

# **Constant Propagation w/ SSA- and Predicated SSA Form**

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**This is joint work with Oliver Rüthing**



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## Outline of the Talk

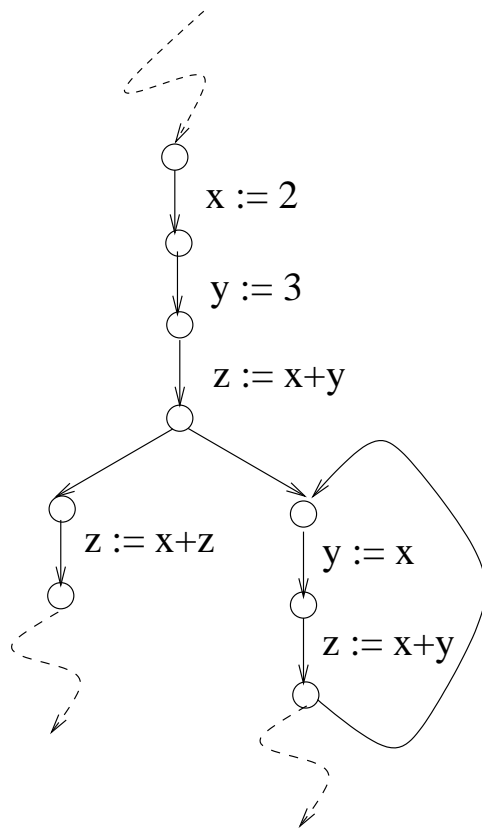
- Part I: Constant Propagation
- Part II: Constant Propagation w/ SSA Form
- Part III: Constant Propagation w/ Predicated SSA Form

# **Part I: Constant Propagation**

# Constant Propagation

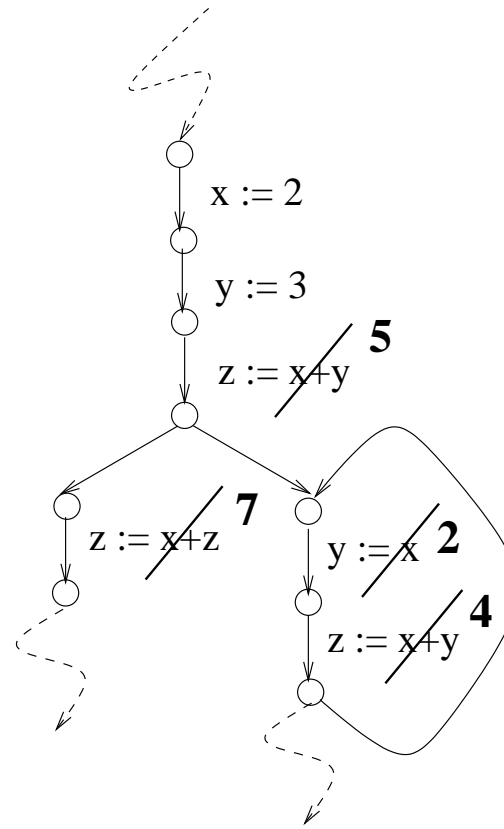
The very idea...

a)



Original program

b)



After simple constant propagation

# Constant Propagation Reconsidered

Remember

- Kildall's algorithm for **simple constants (SC)** (POPL'73)

and Kenneth's talk on Monday morning on further attacks...

- Wegbreit (1st attack)
- Lewis, Tarjan, and Reif (2nd attack)
- Wegman and Zadeck (3rd attack)
- ...

## Constant Propagation Reconsidered (Cont'd)

Advancements of Kildall's work on SC aimed at...

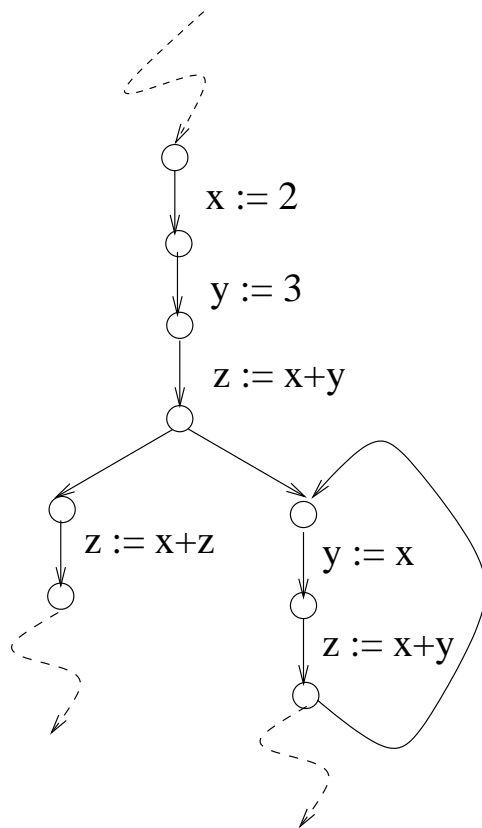
- **Scope**
  - **Interprocedurally**
    - Callahan, Cooper, Kennedy, Torczon (SCC'86)
    - Grove, Torczon (PLDI'93)
    - Metzer, Stroud (LOPLAS, 1993)
    - Sagiv, Reps, Horwitz (TAPSOFT'95)
    - Duesterwald, Gupta, Soffa (TOPLAS, 1997)
  - **Explicitly parallel**
    - Lee, Midkiff, Padua (J. of Parallel Prog., 1998)
    - Knoop (Euro-Par'98)

## Constant Propagation Reconsidered (Cont'd)

- Performance
  - **SSA**: Wegman, Zadeck (POPL'85)
- Expressivity
  - “SC+”: Kam, Ullman (Acta Inf., 1977)
  - **Conditional Constants**: Wegman, Zadeck (POPL'85)
  - **Finite Constants**: Steffen, Knoop (MFCS'89)

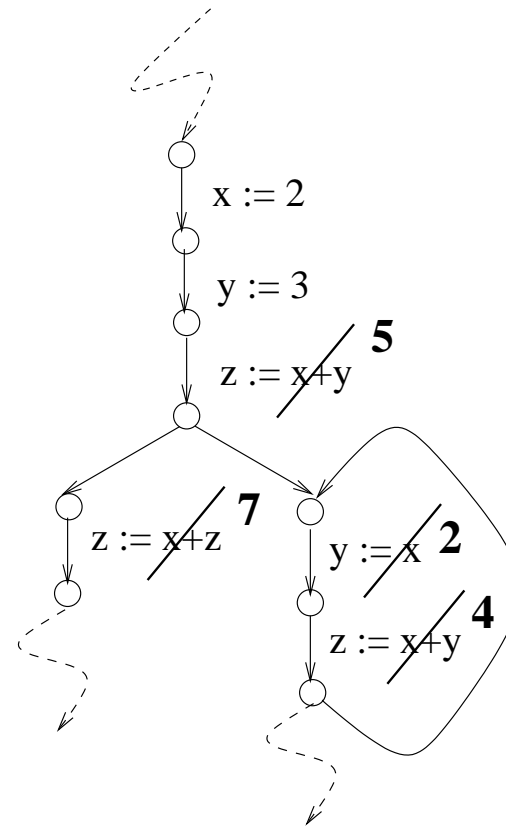
# Why Striving for Greater Expressivity?

a)



Original program

b)



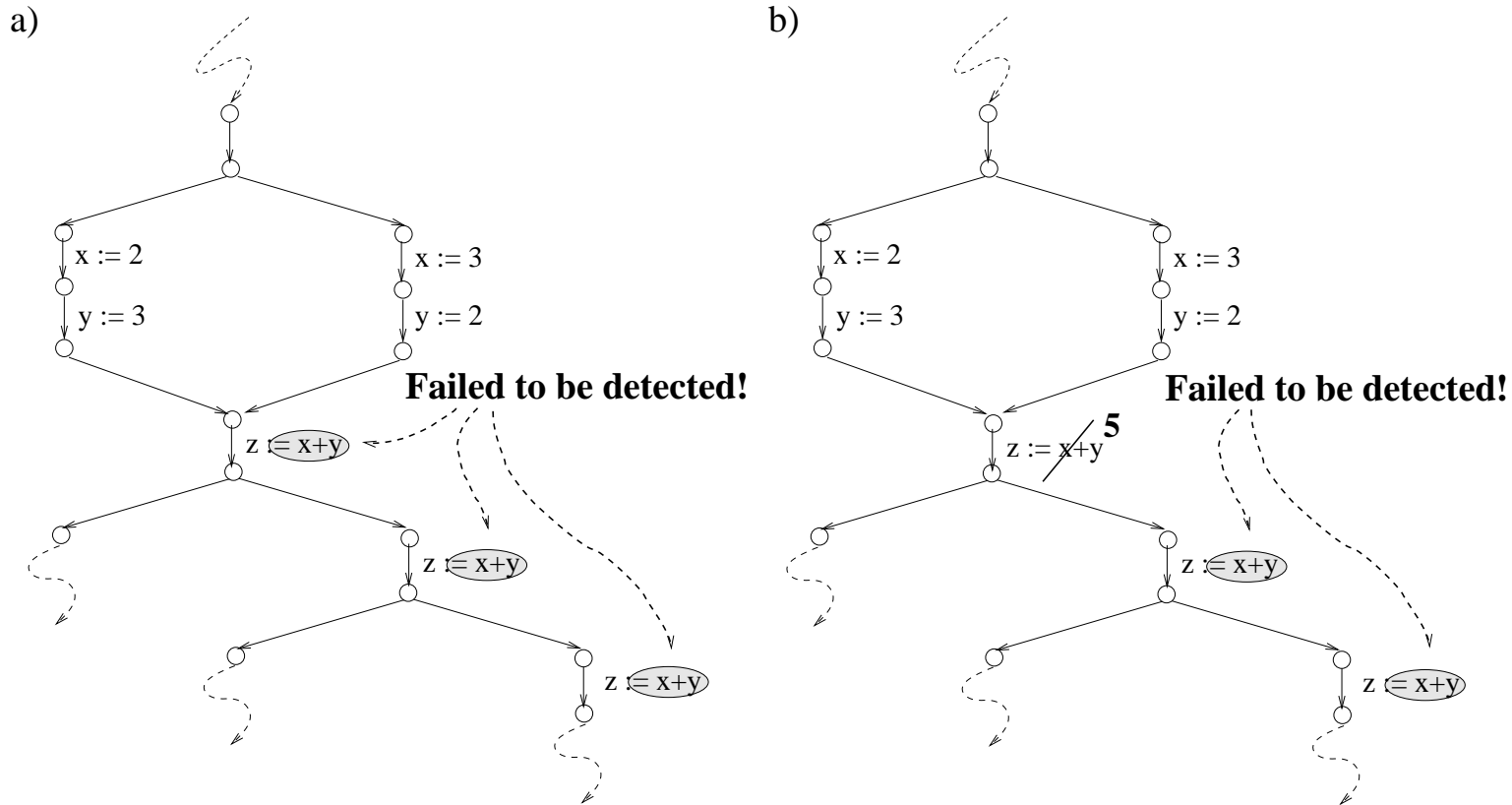
After simple constant propagation

**It's ok, isn't it?**



# Actually, it is not

Simple constants are weak...



After simple constant propagation  
(Note: No effect at all!)

After simple constant propagation  
enhanced by the "look-ahead-of-one" heuristics  
of Kam and Ullman

## Decidability Issues of Constant Propagation

As a matter of fact...

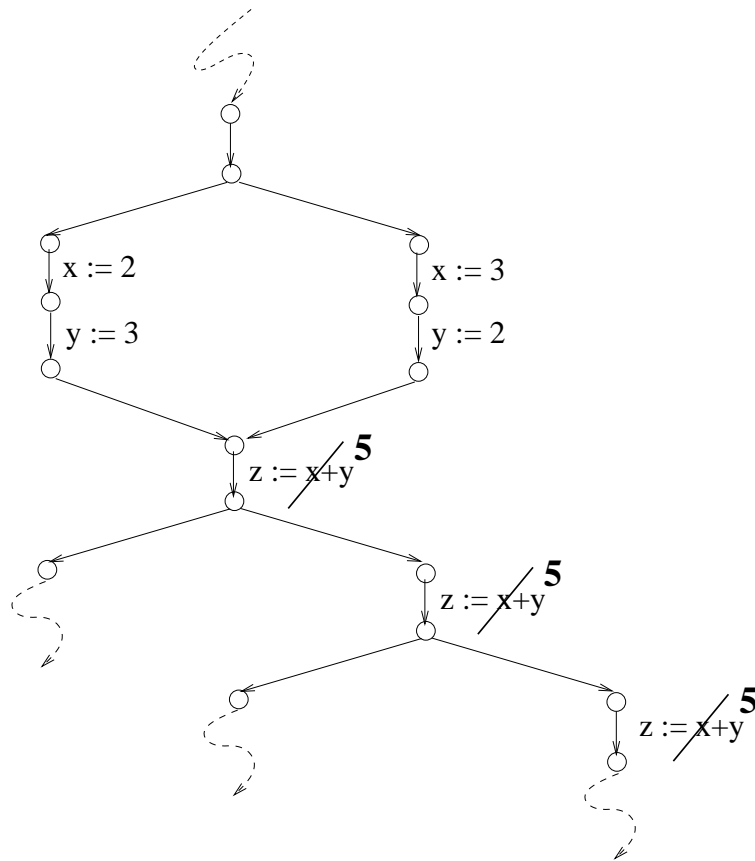
- Constant propagation is undecidable

On the other hand...

- Constant propagation is decidable on DAGs

# Finite Constants (FC)

...are **optimal** on DAGs!



## Finite Constants (Cont'd)

Intuitively

- FC are a systematic, exhaustive, and finitely computable extension of Kam&Ullman's "look-ahead of one" heuristics

Key Facts on Finite Constants

- Proper extension of SC for unrestricted control flow
- Optimal on DAGs
- Exponential worst-case time complexity (even on DAGs)

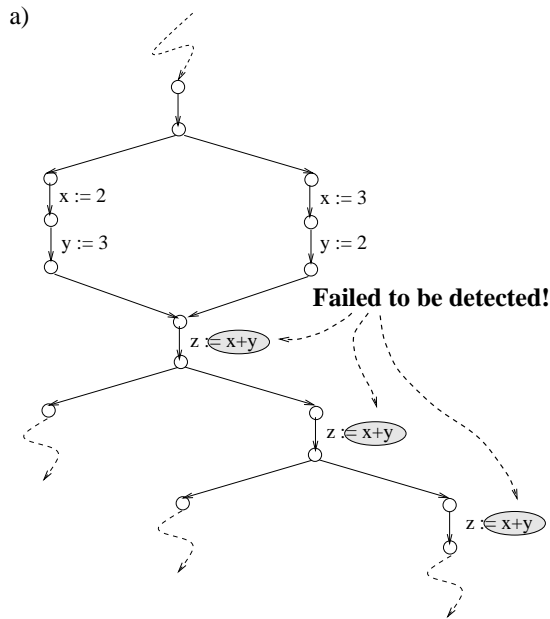
## Note

- Constant Propagation on DAGs is **Co-NP-Complete**

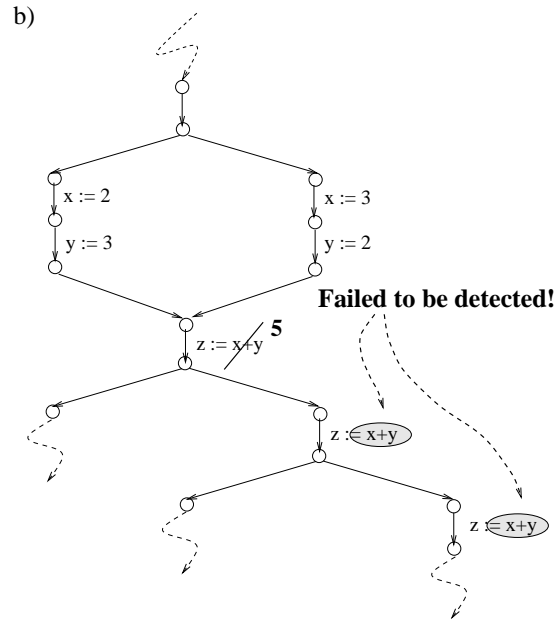
Knoop, Rüthing (CC'00)

Müller-Olm, Rüthing (ESOP'01)

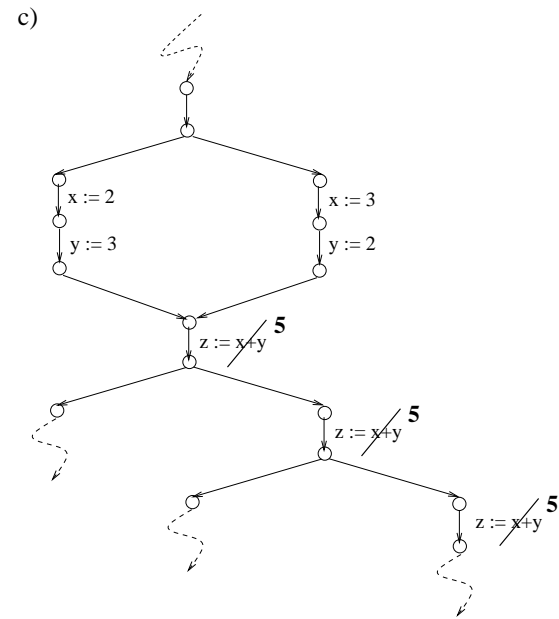
# Reconsidering the Running Example



After simple constant propagation  
(Note: No effect at all!)



After simple constant propagation  
enriched by the "look-ahead-of-one" heuristics  
of Kam and Ullman



The effect of the new algorithm

## A New CP Algorithm

...carefully balancing

- Expressivity and Performance

This new algorithm is...

- based on the **Value Graph** of Alpern, Wegman, and Zadeck (POPL'88)
- which itself is based on **SSA**

Hence: **CP w/ SSA form** (instead of on SSA form)

## **Part II: Constant Propagation w/ SSA Form**

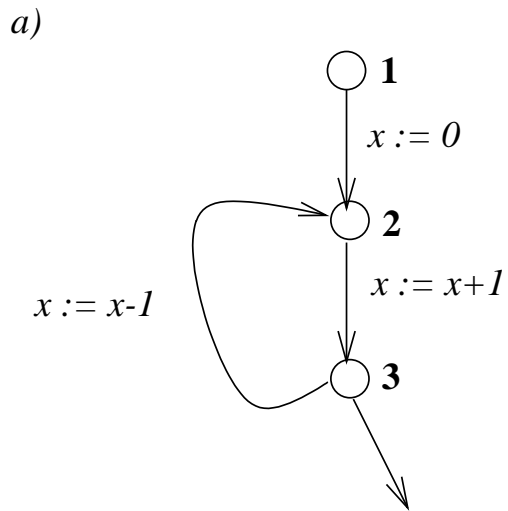


## Own Work Related to Part II of the Talk

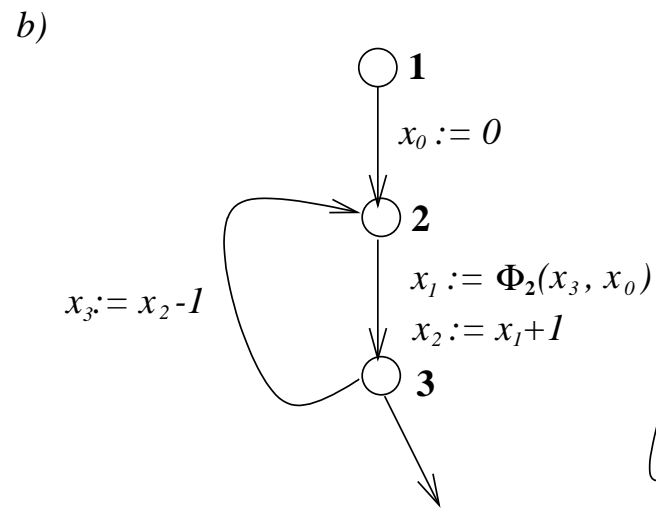
Joint work with Oliver Rüthing...

- Constant Propagation **w/ SSA Form**
  - *Constant Propagation on the Value Graph: Simple Constants and Beyond*. In Proc. 9th Int. Conf. on Compiler Construction (CC 2000), LNCS 1781 (2000), 94 - 109.

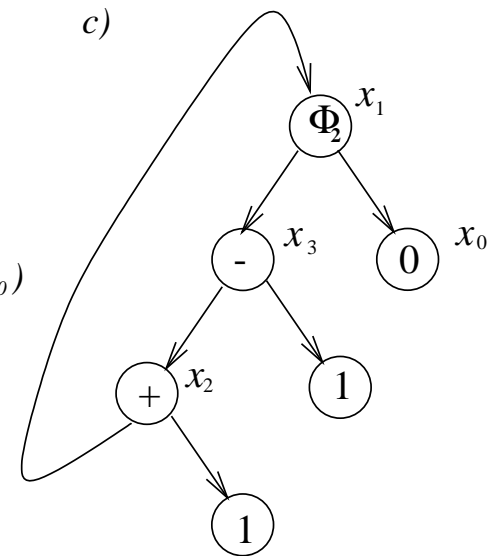
# The Value Graph of Alpern, Wegman, and Zadeck



Original Program

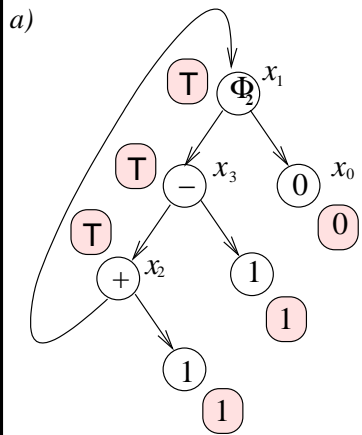


SSA Form

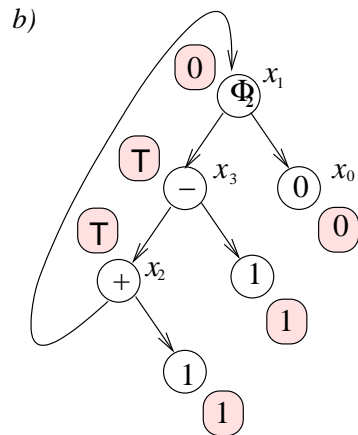


Value Graph

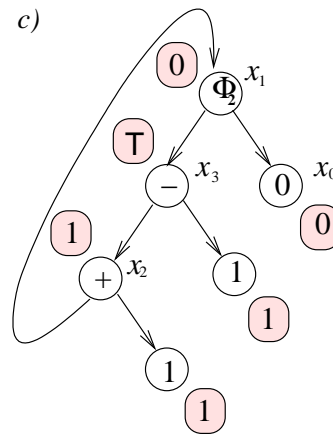
# Constant Propagation on the Value Graph



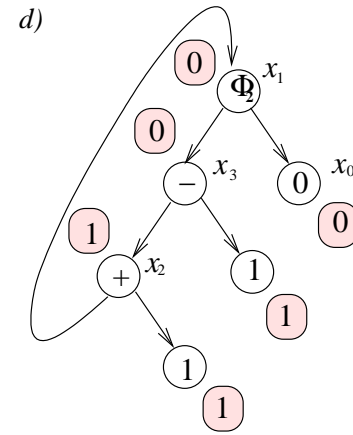
After the initialization step



After the 1st iteration step



After the 2nd iteration step



After the 3rd iteration step: Stable

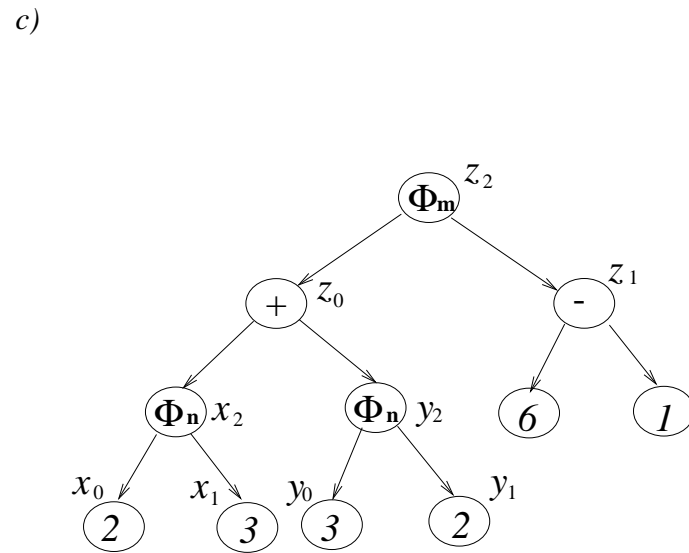
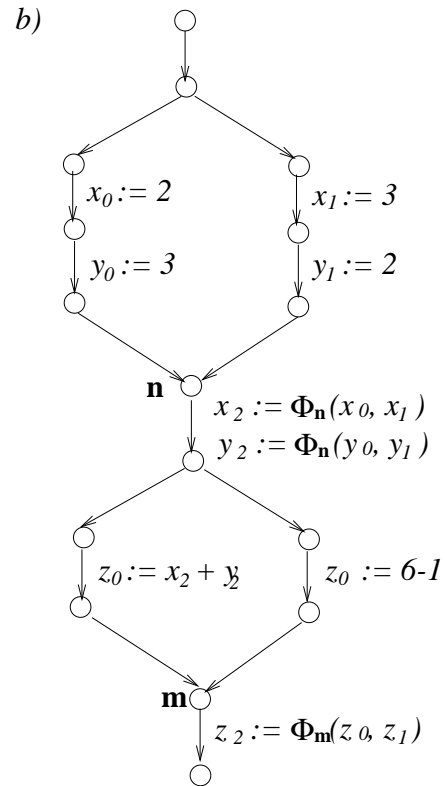
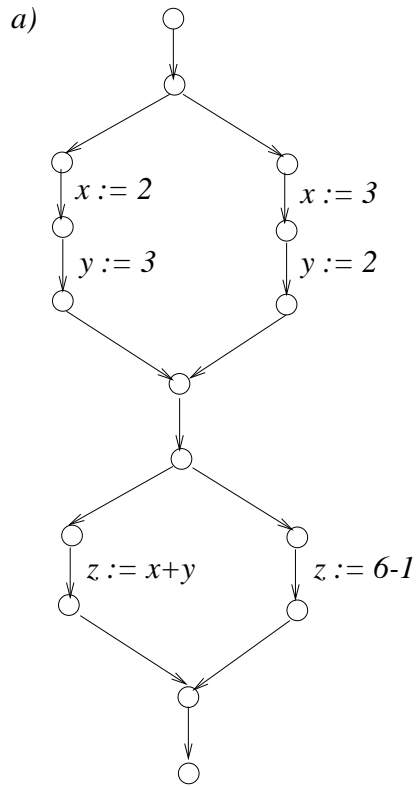
Hence:  $x_2$  and  $x_3$  have constant values!

## Constant Propagation on the Value Graph

...comes in two flavours

- The Basic Algorithm  
...computes SC
- The Full Algorithm  
...goes beyond and integrates the look-ahead heuristics

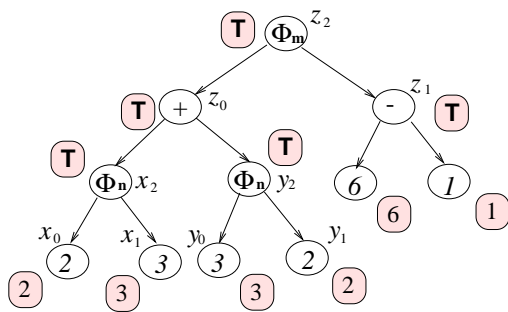
# A New Example for Illustrating the Full Algorithm



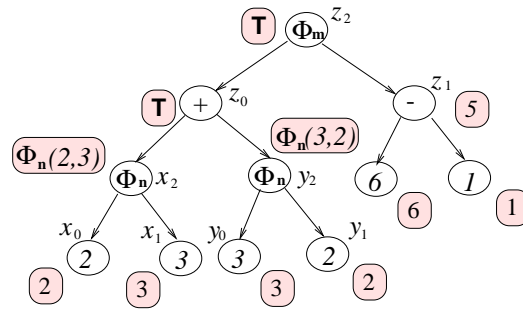
Original Program SSA Form

Value Graph

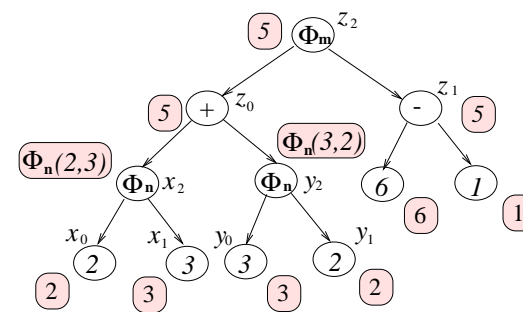
# The Full Algorithm on the Value Graph



The start annotation



After the first iteration



After the second iteration. Stable!

**Clou:** Introducing  $\Phi$ -Constants and  
Adapting the Evaluation Function on Value Graphs!

## Main Results

### Unrestricted Control-Flow...

- The full algorithm detects a superset of SC (even constants, which are no finite constants!)

### Acyclic Control-Flow...

- The full algorithm detects every constant, which is only composed of operators, which are injective in their relevant arguments

### Overall...

- Nicely balances expressivity and performance
- **SSA** and the **Value Graph** are key

## **Part III: Constant Propagation w/ Predicated SSA Form**

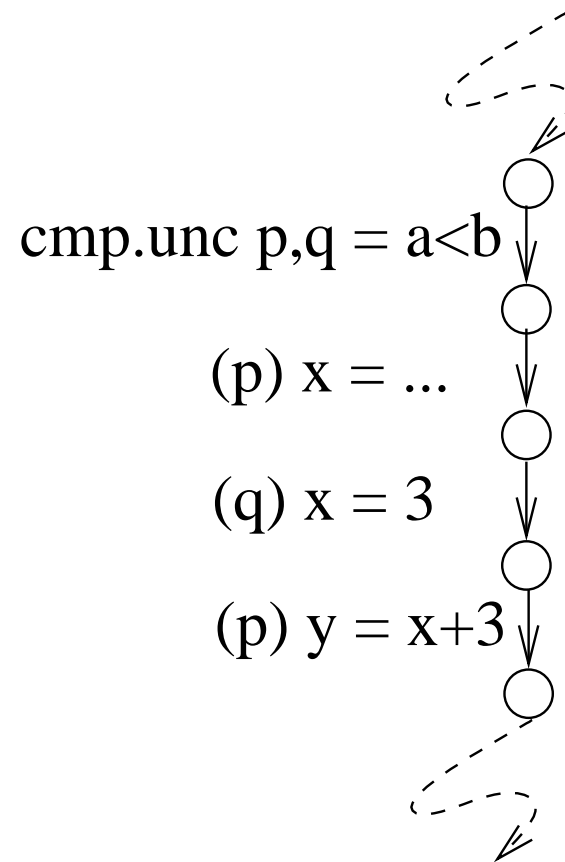


## Own Work Related to Part III of the Talk

Joint work with Oliver Rüthing...

- Constant Propagation **w/ Predicated SSA Form**
  - *Constant Propagation on Predicated Code*. J. of Universal Computer Science 9, 8 (2003), 829 - 850. (special issue devoted to SBLP'03).
  - *Constant Propagation on Predicated Code*. In Proc. 7th Brazilian Symp. on Programming Languages (SBLP 2003), 135 - 148.

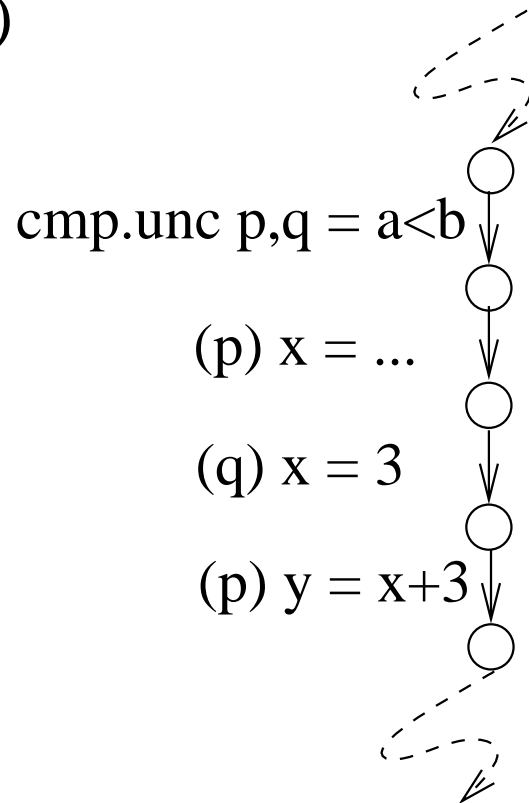
## Predicated Code



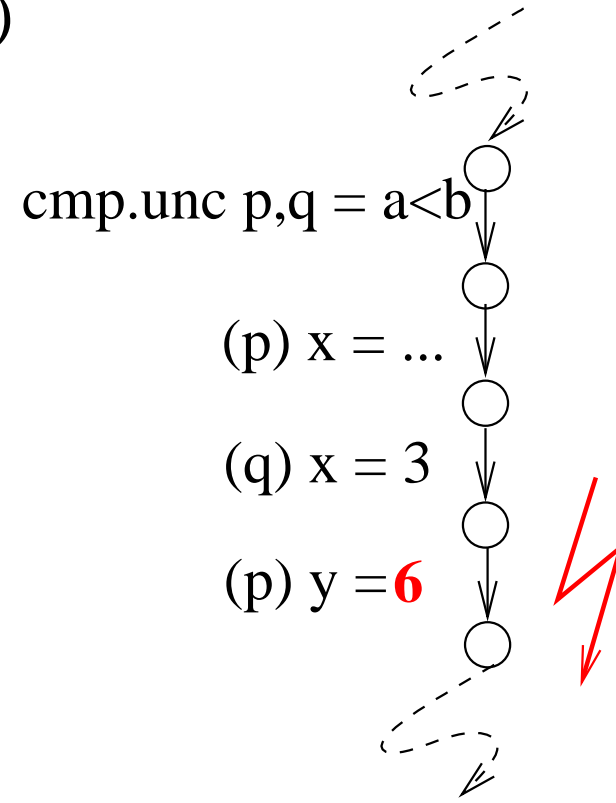
...resulting from if-conversion.

# Performing CP Naively on Predicated Code Fails...

a)

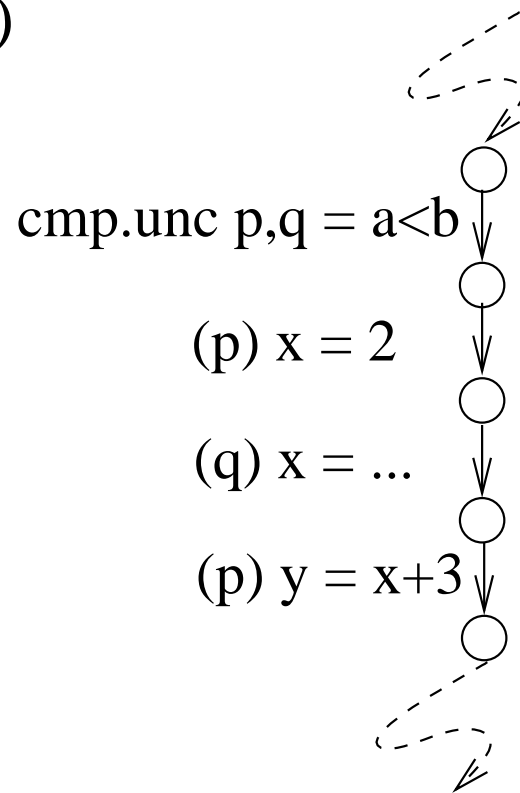


b)

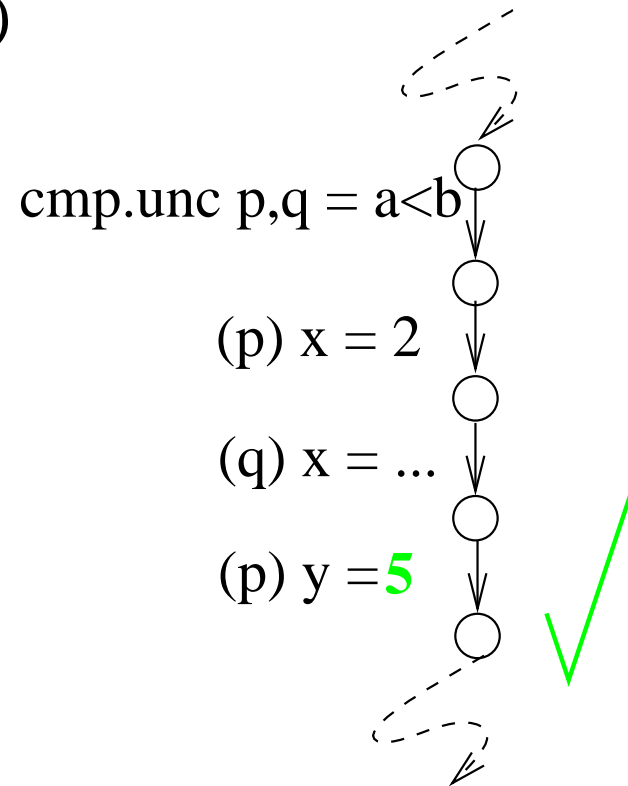


## On the Other Hand...

a)



b)



...naive sound CP is likely to be too conservative and to miss many optimization opportunities!

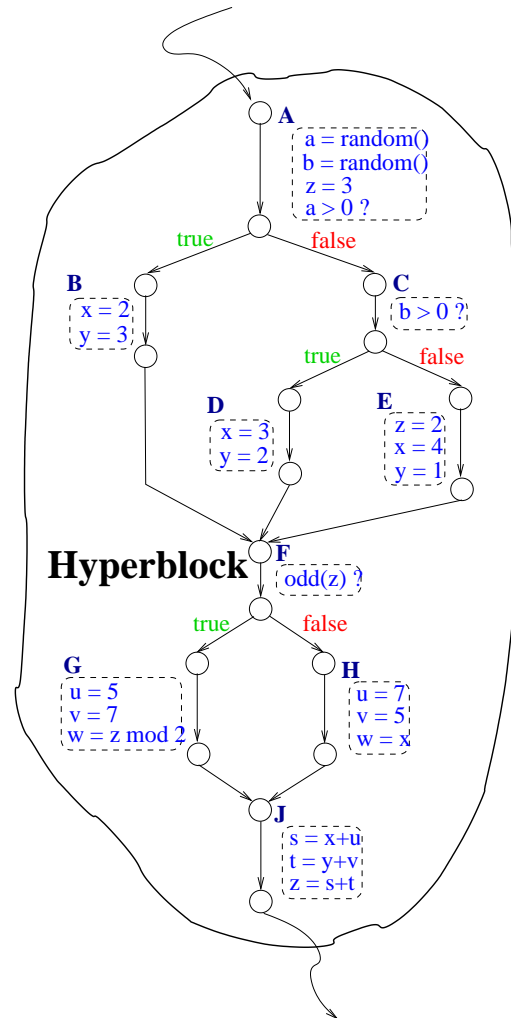
# **Workplan: Handling Predicated Code more Smartly**

## **Hyperblocks**

...important building blocks in predicated code.

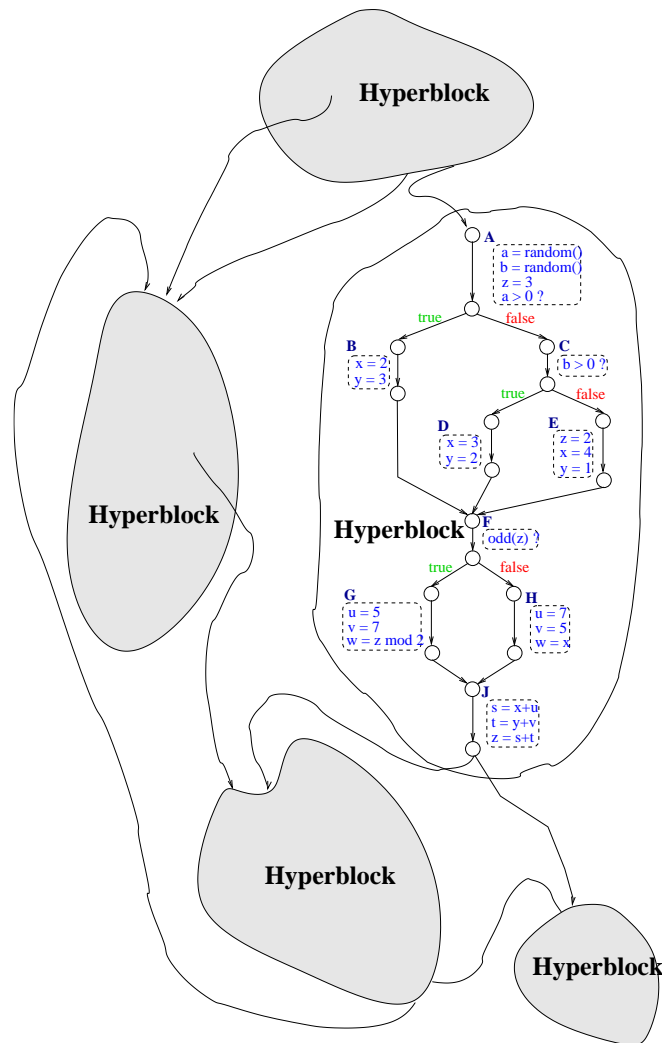
# A Hyperblock

Single entry, multiple exits...



# Embedded into a Program

The running example...



# The New CP Algorithm on Predicated Code

...comes in two/**plus** flavours

- The Basic Algorithm
- The Full Algorithm

**plus**

- Performance-tuned Variants

Each consisting of a

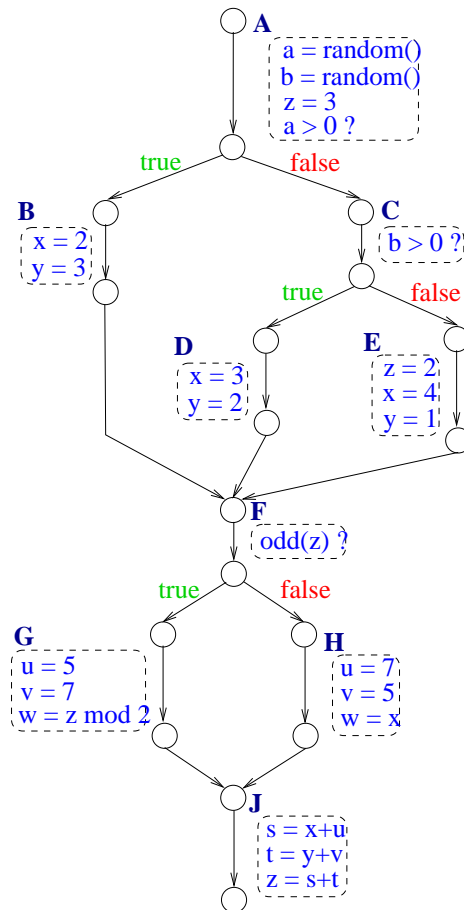
- global
- local

stage.



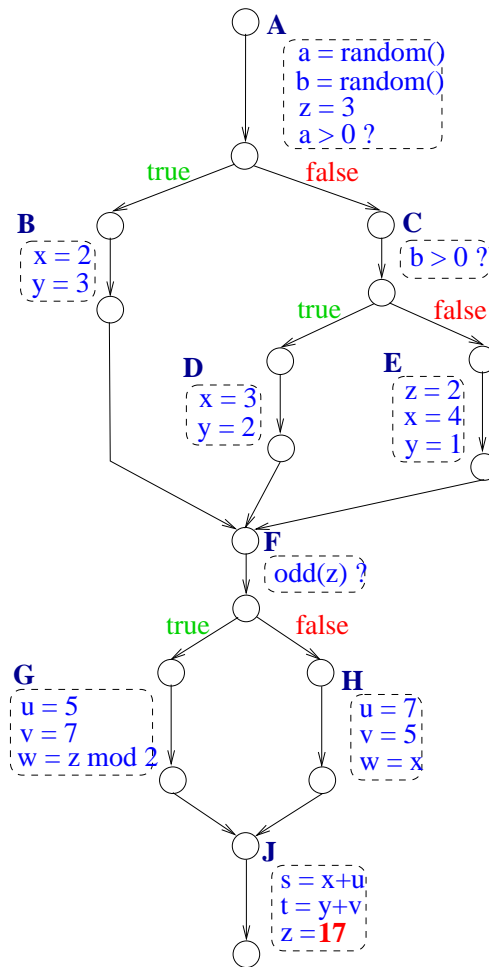
# Discussing the Local Stage

The **hyperblock** we will focus on...



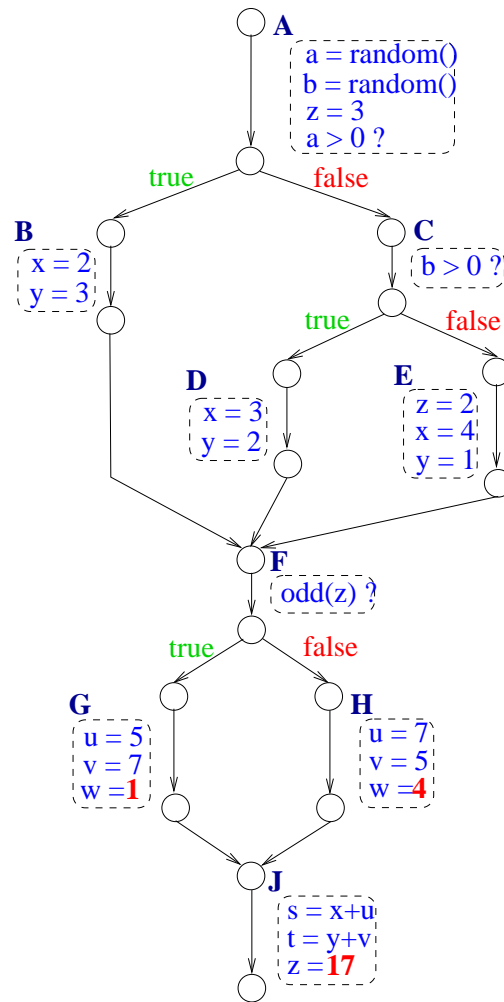
Original Hyperblock

# Optimization of the Basic Algorithm



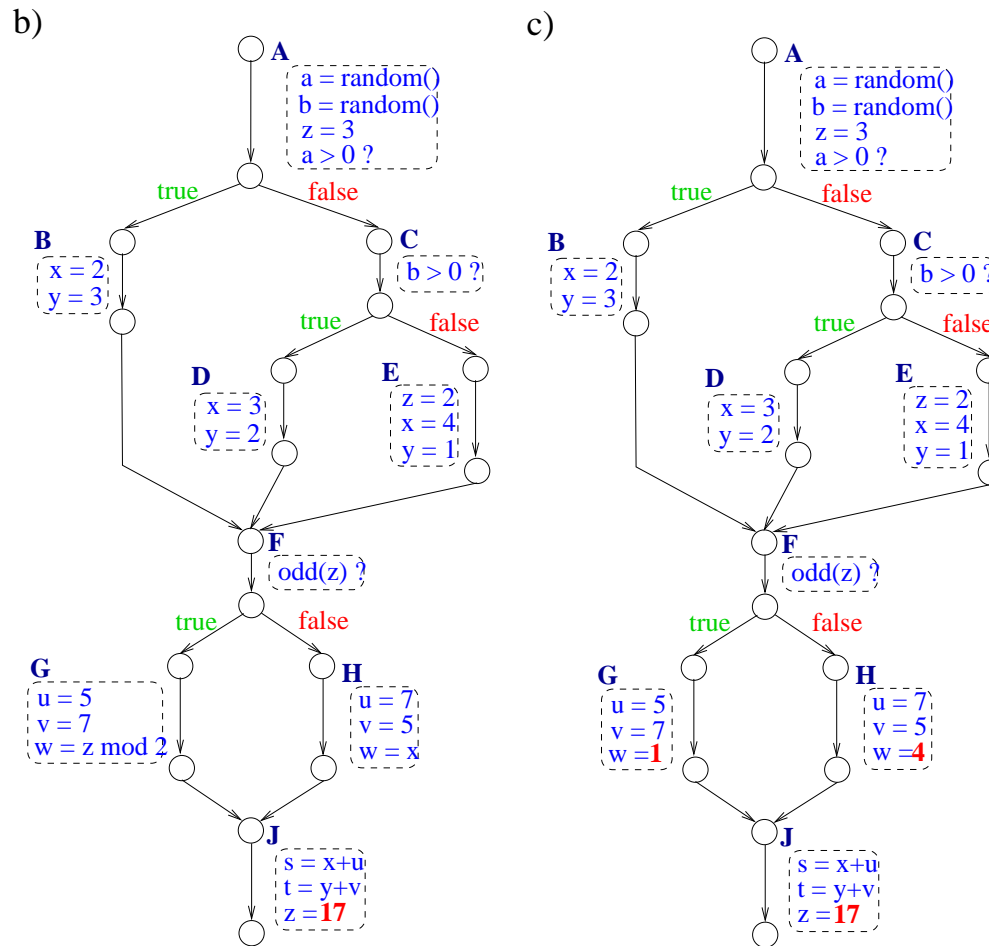
The Non-Deterministic Path-Precise  
Basic Optimization

# Optimization of the Full Algorithm



The Deterministic Path-Precise  
Full Optimization

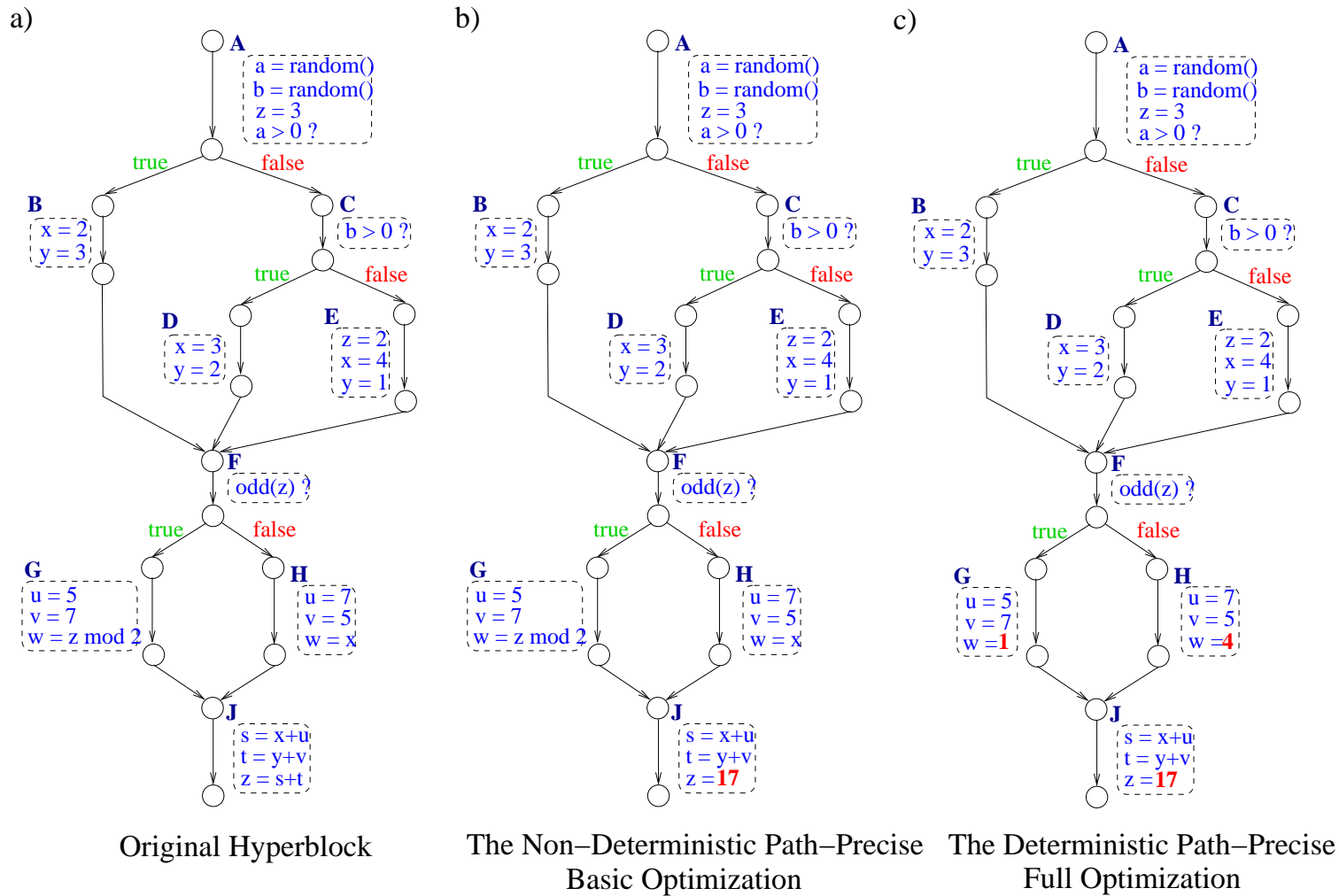
# Optimizations of Basic and Full Alg. at a Glance



The Non-Deterministic Path-Precise  
Basic Optimization

The Deterministic Path-Precise  
Full Optimization

# Optimizations of Basic and Full Alg. at a Glance



## Original and Predicated Code

<code>begin \\ Original Hyperblock</code>	<code>begin \\ After if-Conversion</code>
<code>(a,b) = (random(),random());</code>	<code>(p0) (a,b) = (random(),random());</code>
<code>z = 3;</code>	<code>(p0) z = 3;</code>
<code>if a&gt;0 then</code>	<code>(p0) cmp.unc B,C (a&gt;0);</code>
<code>  x = 2;</code>	<code>(B) x = 2;</code>
<code>  y = 3</code>	<code>(B) y = 3;</code>
<code>elseif b&gt;0 then</code>	<code>(C) cmp.unc D,E (b&gt;0);</code>
<code>  x = 3;</code>	<code>(D) x = 3;</code>
<code>  y = 2</code>	<code>(D) y = 2;</code>
<code>else</code>	<code>(E) z = 2;</code>
<code>  z = 2;</code>	<code>(E) x = 4;</code>
<code>  x = 4;</code>	<code>(E) y = 1;</code>
<code>  y = 1 fi;</code>	<code>(p0) cmp.unc G,H (odd(z));</code>
<code>if odd(z) then</code>	<code>(G) u = 5;</code>
<code>  u = 5;</code>	<code>(G) v = 7;</code>
<code>  v = 7;</code>	<code>(G) w = z mod 2;</code>
<code>  w = z mod 2</code>	<code>(H) u = 7;</code>
<code>else</code>	<code>(H) v = 5;</code>
<code>  u = 7;</code>	<code>(H) w = x;</code>
<code>  v = 5;</code>	<code>(p0) s = x+u;</code>
<code>  w = x fi;</code>	<code>(p0) t = y+v;</code>
<code>s = x+u;</code>	<code>(p0) z = s+t end.</code>
<code>t = y+v;</code>	
<code>z = s+t end.</code>	

## **Predicated SSA**

...by Carter, Simon, Calder, Ferrante (PACT'99)

```

begin (p0)      A = OR(TRUE);           | [*] (HFBA)    w2 = x1;
  (A)          (a1,b1) = (random(),random()); | [*] (HFDCA)   w2 = x2;
  (A)          z1 = 3;                   |      (HFECA)  w2 = x3;
  (A)          cmp.unc BA,CA (a1>0);      |      (H)      u2 = 7;
  (p0)         B = OR(BA);               |      (H)      v2 = 5;
  (p0)         C = OR(CA);               |      (GFBA)   JGFBA = OR(TRUE);
  (B)          x1 = 2;                   |      (GFDCA)  JGFDCA = OR(TRUE);
  (B)          y1 = 3;                   | [*] (GFECA)  JGFECA = OR(TRUE);
  (C)          cmp.unc DCA,ECA (b1>0);    | [*] (HFBA)   JHFBA = OR(TRUE);
  (p0)         D = OR(DCA);              | [*] (HFDCA)  JHFDCA = OR(TRUE);
  (p0)         E = OR(ECA);              |      (HFECA)  JHFECA = OR(TRUE);
  (D)          x2 = 3;                   | [-] (p0)     J = OR(JGFBA,JGFDCA,
  (D)          y2 = 2;                   |                                     JGFECA,JHFBA,
  (E)          z2 = 2;                   |                                     JHFDCA,JHFECA);
  (E)          x3 = 4;                   |      (JGFBA)  s1 = x1+u1;
  (E)          y3 = 1;                   |      (JGFBA)  t1 = y1+v1;
  (BA)         FBA = OR(TRUE);           | [*] (JGFDCA) s1 = x2+u1;
  (DCA)        FDCA = OR(TRUE);          | [*] (JGFDCA) t1 = y2+v1;
  (ECA)        FECA = OR(TRUE);          |      (JGFECA) s1 = x3+u1;
  (p0)         F = OR(FBA,FDCA,FECA);    |      (JGFECA) t1 = y3+v1;
  (FBA)        cmp.unc GFBA,HFBA (odd(z1)); | [*] (JHFBA)  s1 = x1+u2;
  (FDCA)       cmp.unc GFDCA,HFDCA (odd(z1)); | [*] (JHFBA)  t1 = y1+v2;
  (FECA)       cmp.unc GFECA,HFECA (odd(z2)); | [*] (JHFDCA) s1 = x2+u2;
  [-] (p0)     G = OR(GFBA,GFDCA,GFECA); | [*] (JHFDCA) t1 = y2+v2;
  [-] (p0)     H = OR(HFBA,HFDCA,HFECA); |      (JHFECA) s1 = x3+u2;
  (GFBA)      w1 = z1 mod 2;             |      (JHFECA) t1 = y3+v2;
  (GFDCA)     w1 = z1 mod 2;             |      (J)      z3 = s1+t1;
  (GFECA)     w1 = z2 mod 2;
  (G)         u1 = 5;
  (G)         v1 = 7;

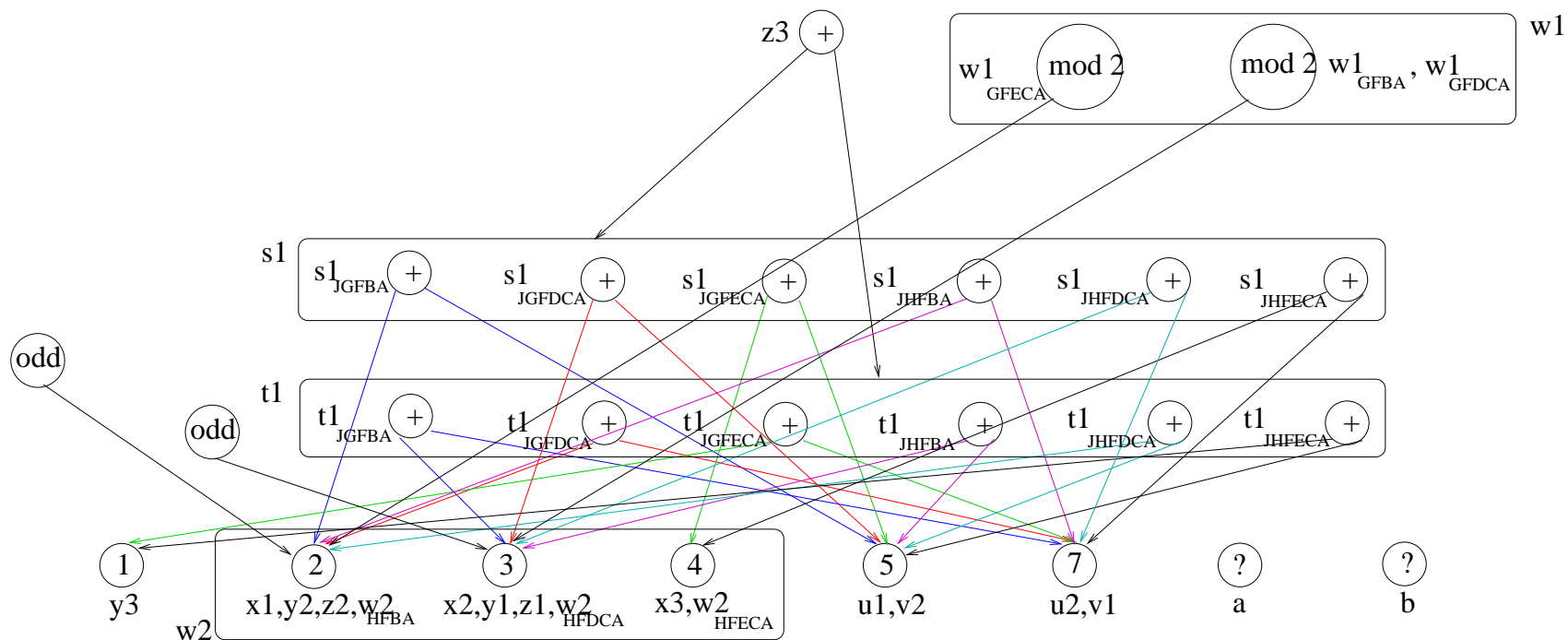
```

```
end.
```

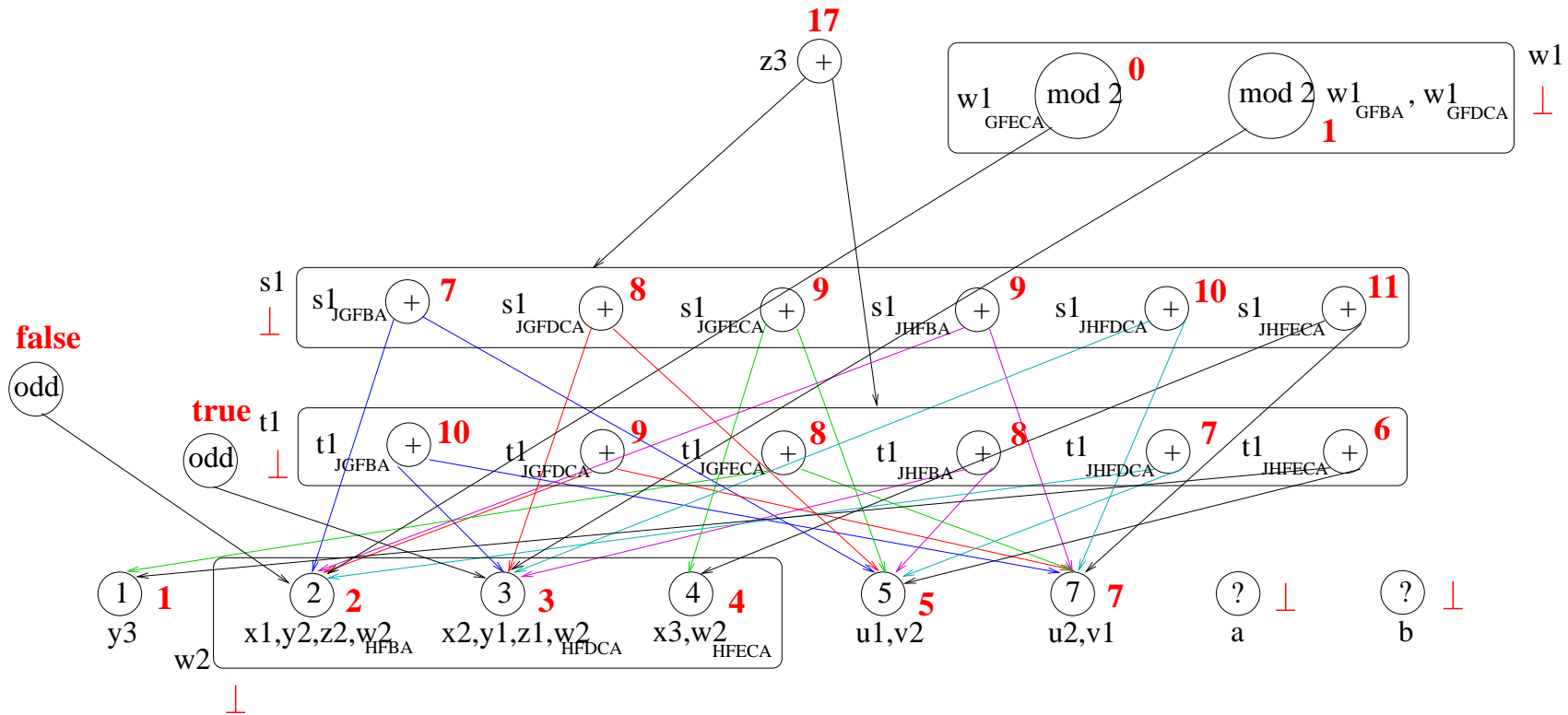


# The Basic Predicated Value Graph based on PSSA Form

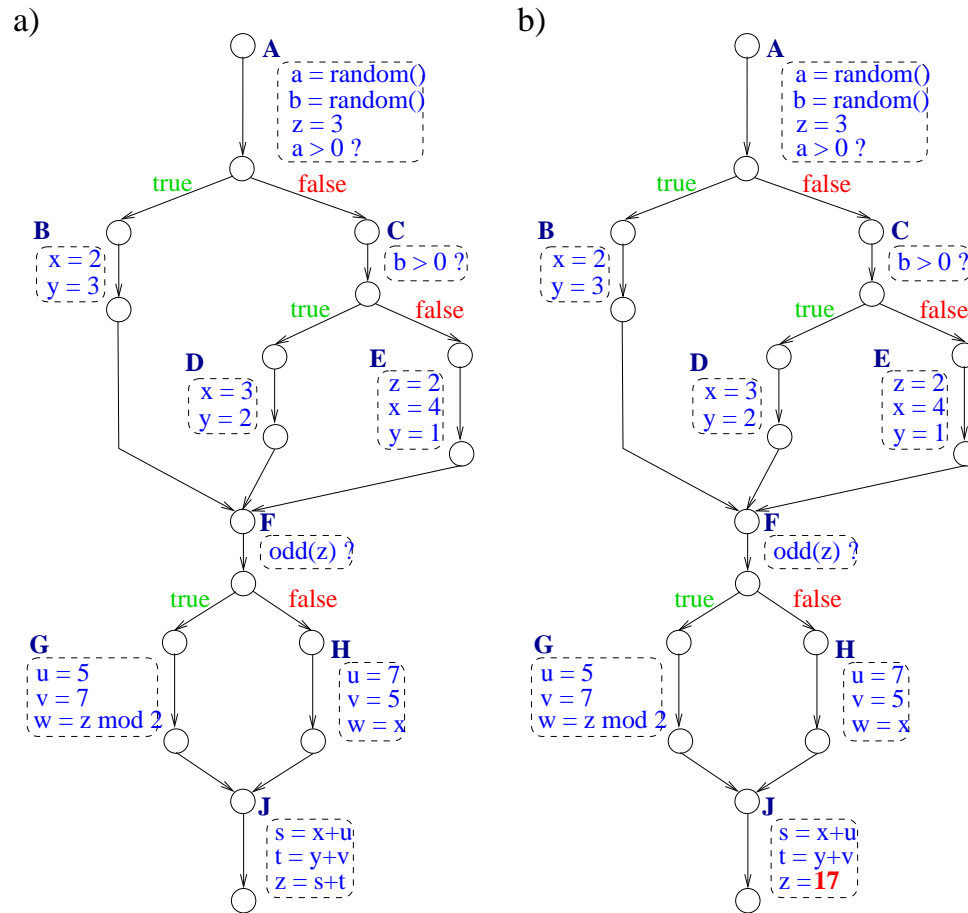
W/out taking advantage of guarding predicates...



# After CP on the Basic PVG / Basic Algorithm



# Optimization of the Basic Algorithm

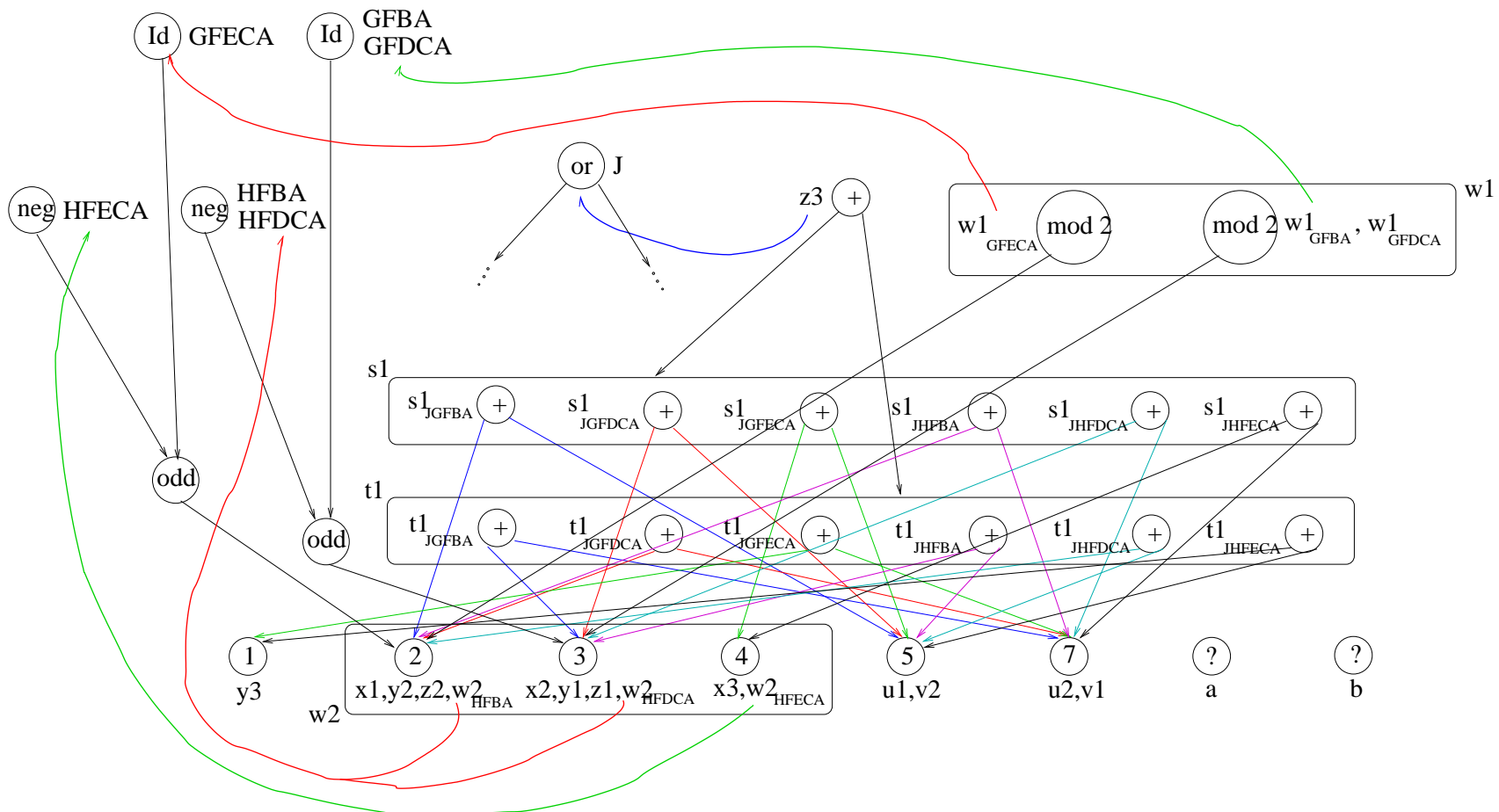


Original Hyperblock

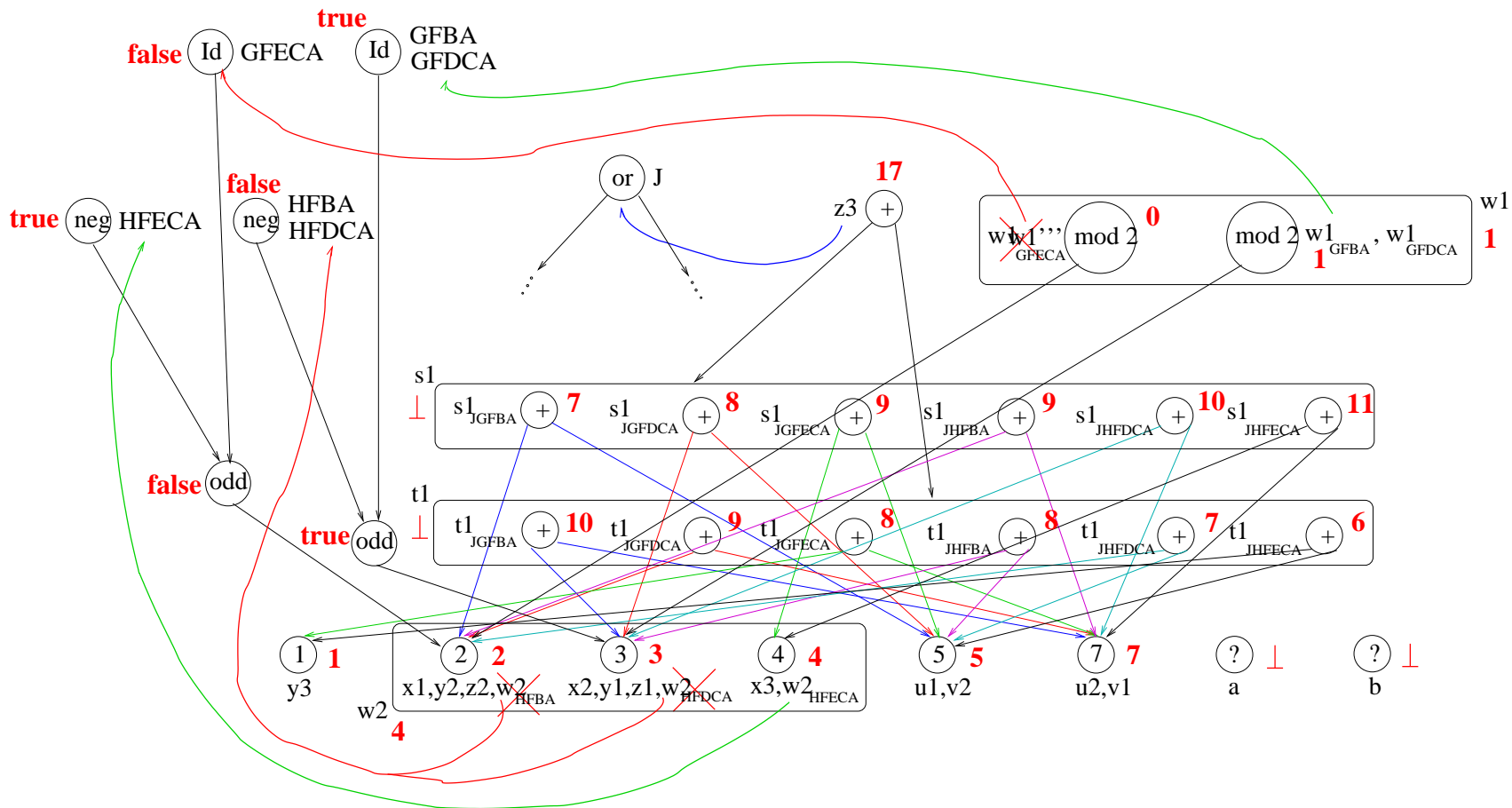
The Non-Deterministic Path-Precise  
Basic Optimization

# The Predicated Value Graph

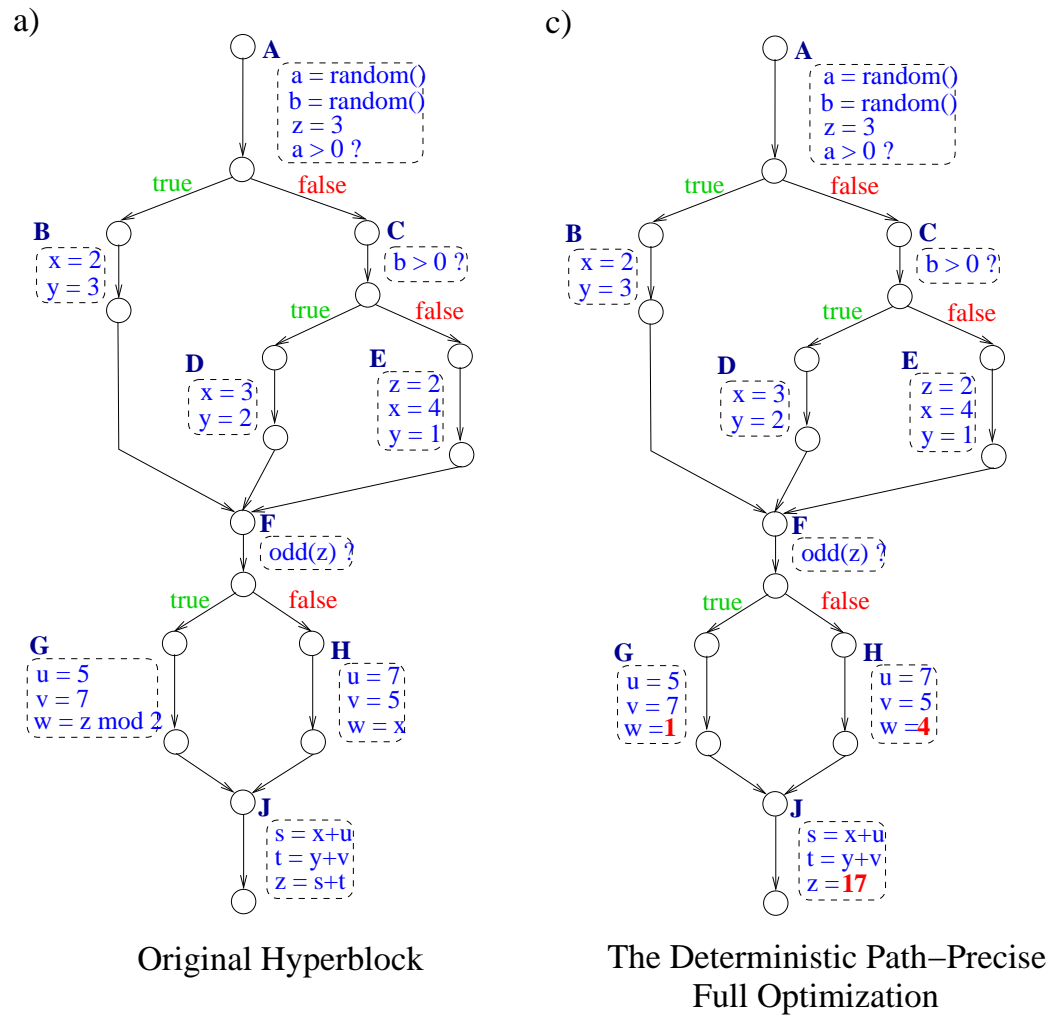
Taking advantage of guarding predicates...



# After CP on the PVG / Full Algorithm



# Optimization of the Full Algorithm



# **The Optimized Hyperblock in PSSA Form**

```

begin (p0)      A = OR(TRUE);
(A)            a1 = random();
(A)            b1 = random();
(A)            z1 = 3;
(A)            cmp.unc BA,CA (a1>0);
(p0)           B = OR(BA);
(p0)           C = OR(CA);
(B)            x1 = 2;
(B)            y1 = 3;
(C)            cmp.unc DCA,ECA (b1>0);
(p0)           D = OR(DCA);
(p0)           E = OR(ECA);
(D)            x2 = 3;
(D)            y2 = 2;
(E)            z2 = 2;
(E)            x3 = 4;
(E)            y3 = 1;
(BA)           FBA = OR(TRUE);
(DCA)          FDCA = OR(TRUE);
(ECA)          FECA = OR(TRUE);
(p0)           F = OR(FBA,FDCA,FECA);
(FBA)          cmp.unc GFBA,HFBA (TRUE));
(FDCA)         cmp.unc GFDCA,HFDCA (TRUE);
(FECA)         cmp.unc GFECA,HFECA (FALSE);
[-] (p0)       G = OR(GFBA,GFDCA);
[-] (p0)       H = OR(HFECA);
(G)            w1 = 1;
(G)            u1 = 5;
(G)            v1 = 7;
(HFECA)        w2 = 4;
(H)            u2 = 7;
(H)            v2 = 5;
(GFBA)         JGFBA = OR(TRUE);
(GFDCA)        JGFDCA = OR(TRUE);
(HFECA)        JHFECA = OR(TRUE);
[-] (p0)       J = OR(JGFBA,JGFECA,
                JHFECA);
(JGFBA)        s1 = 7;
(JGFBA)        t1 = 10;
(JGFECA)       s1 = 9;
(JGFECA)       t1 = 8;
(JHFECA)       s1 = 11;
(JHFECA)       t1 = 6;
(J)            z3 = 17;
end.

```



# Main Results

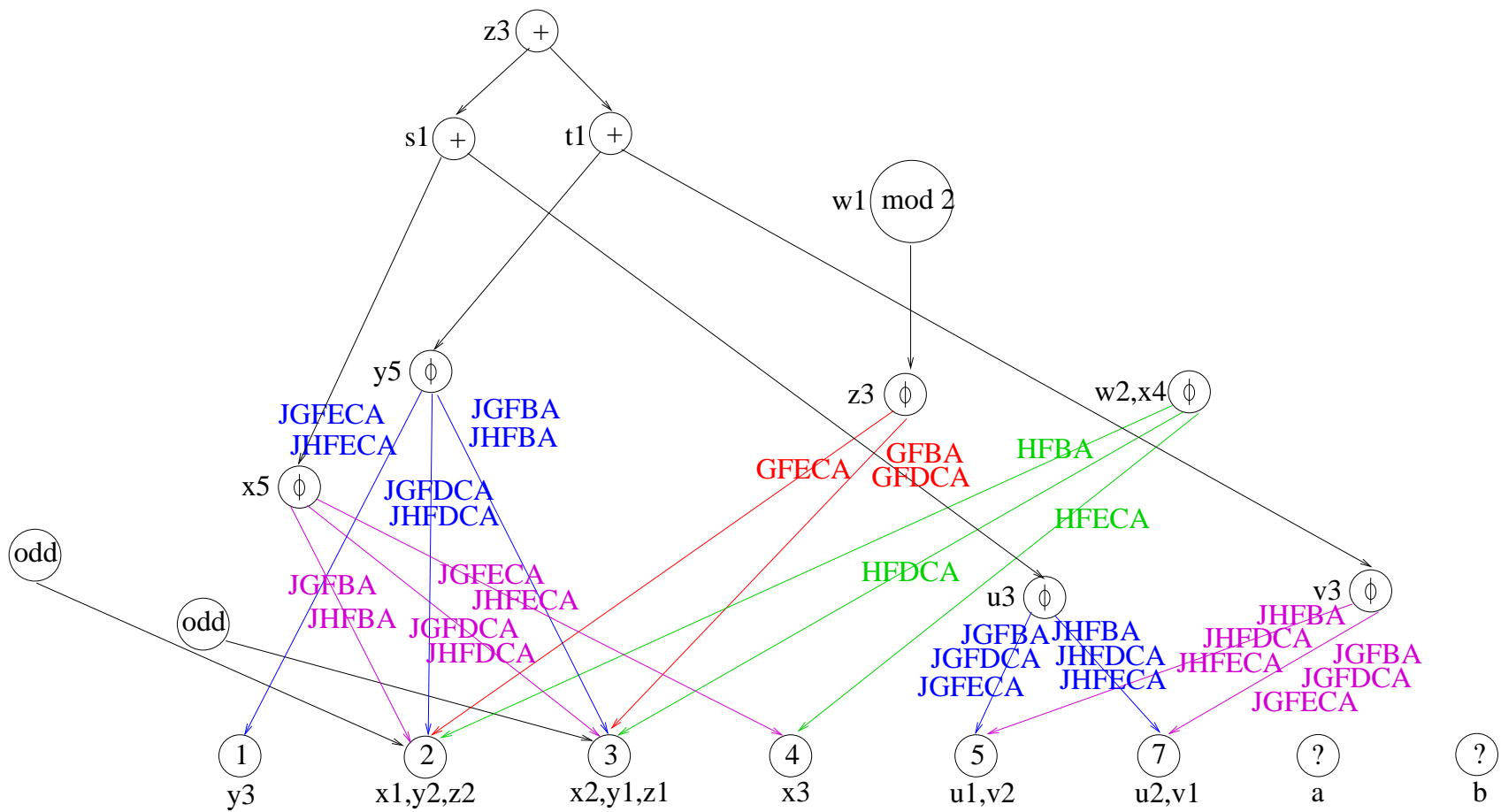
## Soundness

- The global CP-Algorithm is sound (for both the basic and full algorithm of the local stage)

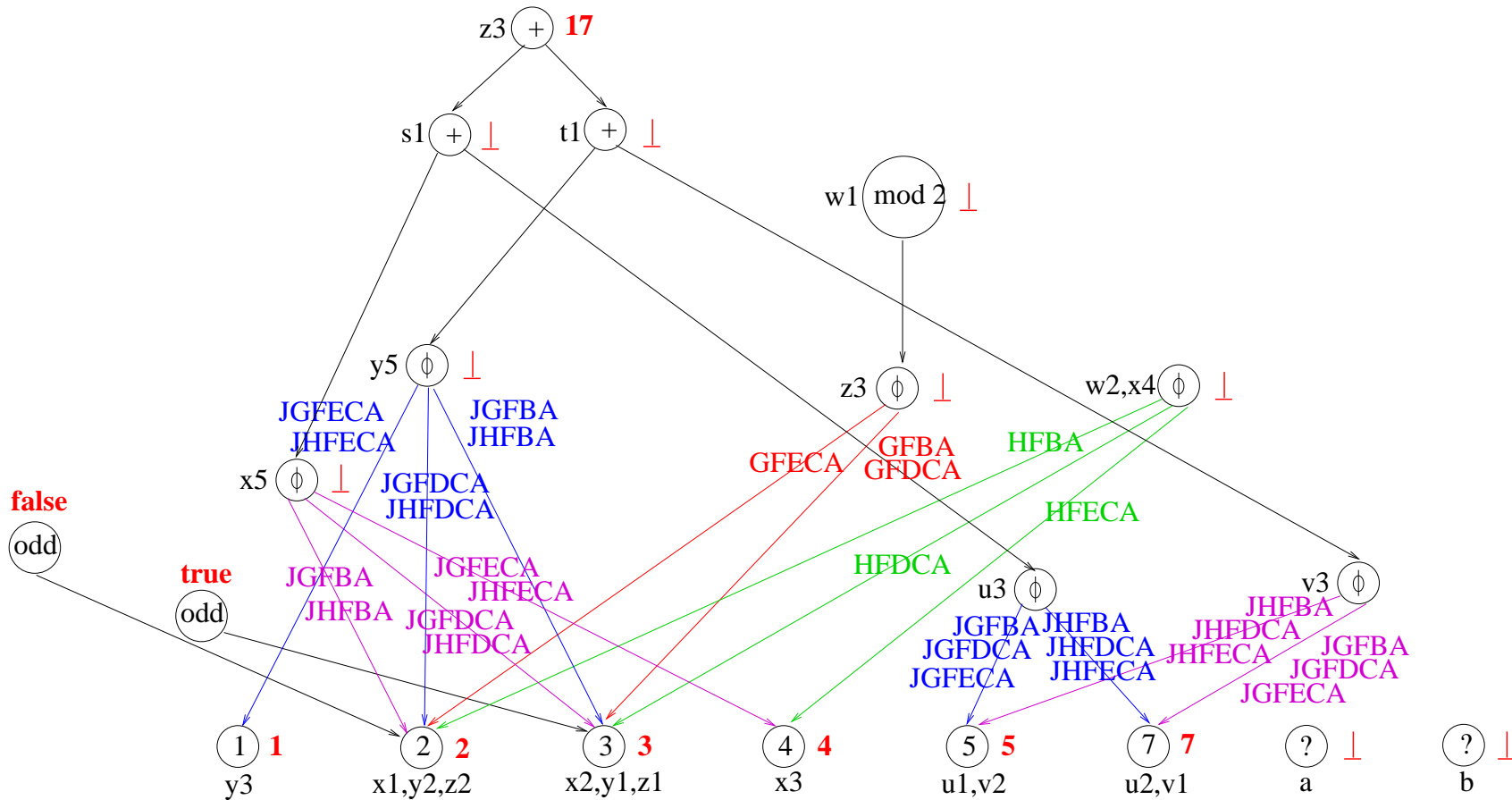
## Completeness/Optimality

- The basic algorithm of the local stage is **trace-precise** wrt non-deterministic interpretation of branches
- The full algorithm of the local stage is **predicate-sensitive trace-precise**

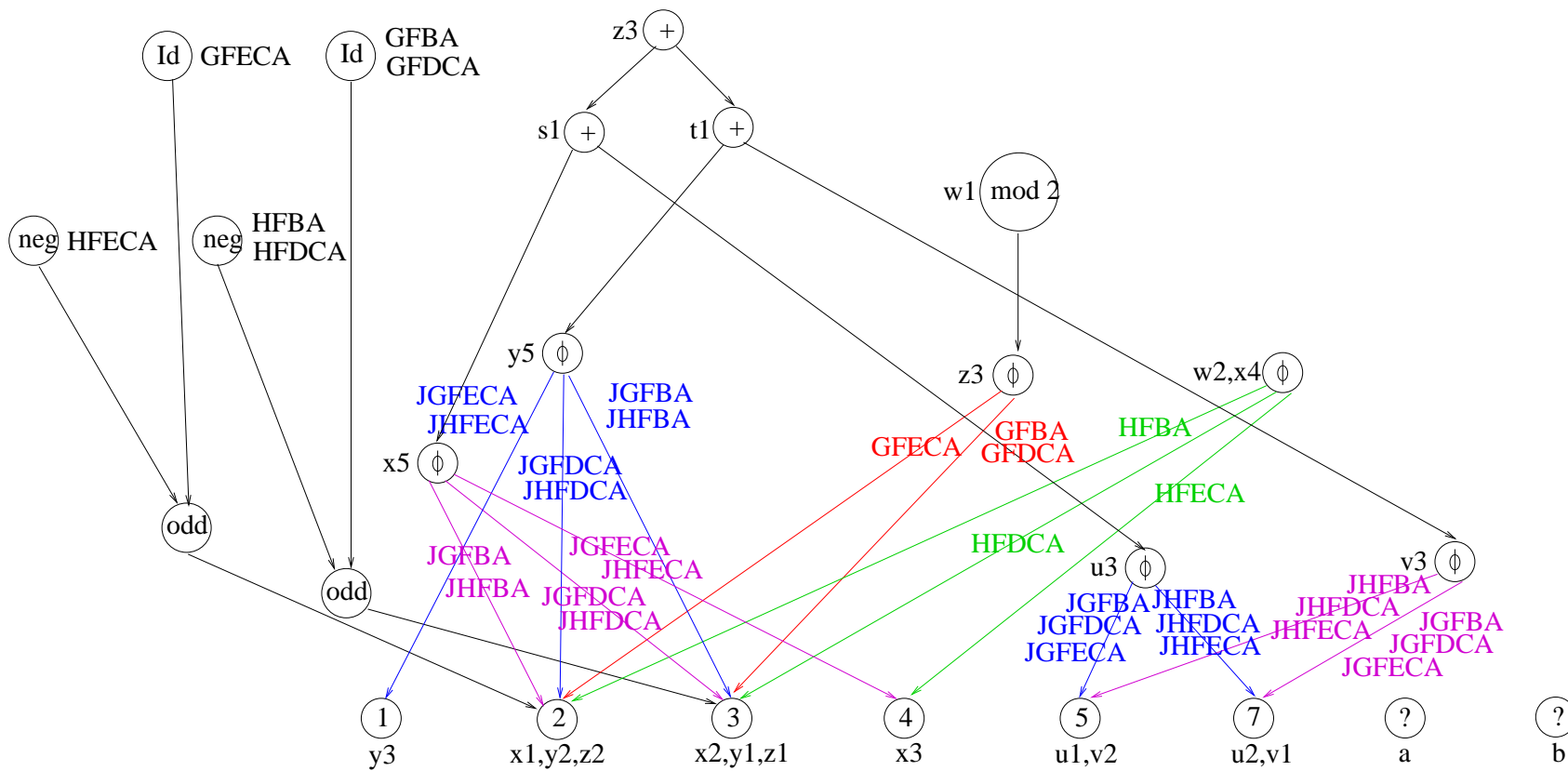
# Tuning the Performance: Basic Algorithm



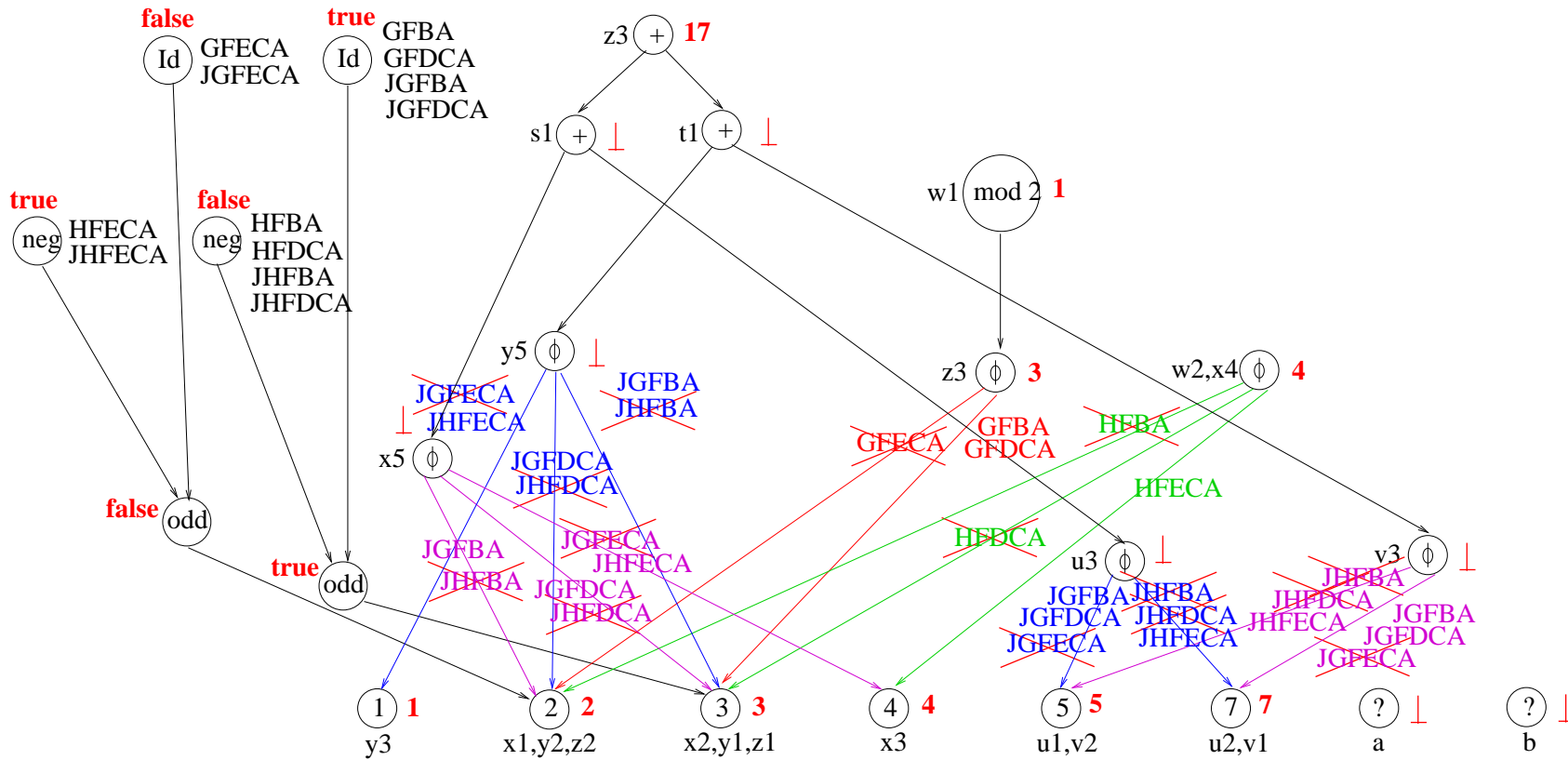
# Tuning the Performance: Basic Alg. (Cont'd)



# Tuning the Performance: Full Algorithm



# Tuning the Performance: Full Alg. (Cont'd)



## Conclusions

### Constant Propagation and SSA/PSSA...

- a perfect match – SSA/PSSA really help!
- Key: Value Graph and Predicated Value Graph

### Open to extensions, e.g.

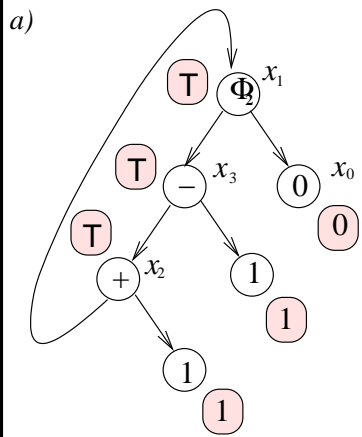
- Value Graph: Conditional Constants

### Overall

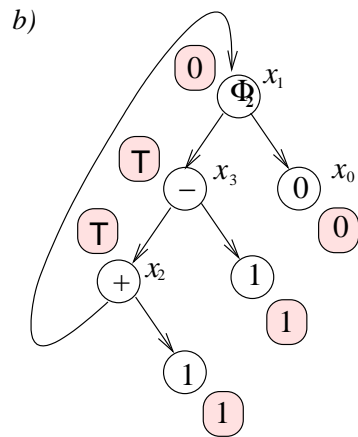
- Especially neat example demonstrating the benefits of SSA

# Constant Propagation w/SSA on the Value Graph

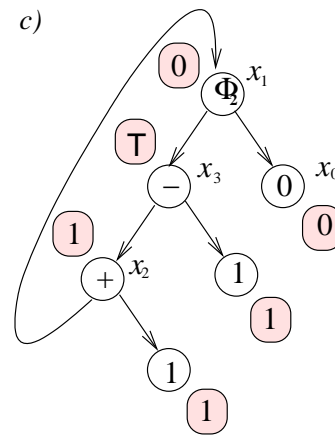
...with **Triple E** Rating: **E**xpressive, **E**fficient, **E**asy!



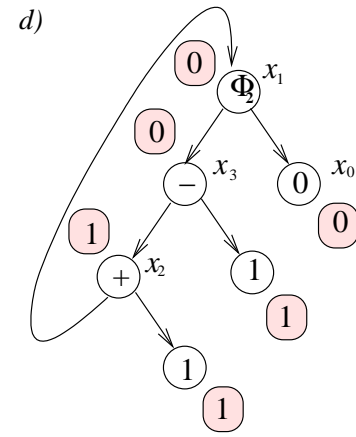
After the initialization step



After the 1st iteration step



After the 2nd iteration step



After the 3rd iteration step: Stable

## Interests – Final Slide

In general...

- programming languages, compilers and everything related to it, especially
  - program analysis, transformation/optimization
  - WCET analysis



## Interests – Final Slide (Cont'd)

Regarding SSA...

SSA & SSA extensions

- as intermediate program representation

and their usage/applications in

- program analysis, transformation/optimization