Attribute Dependencies

– Wilhelm/Seidl/Hack: Compiler Design,
Syntactic and Semantic Analysis –

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Attribute Dependencies

Attribute dependencies

- relate attribute occurrences (and instances),
- describe which attribute occurrences (instances) depend on which other occurrences (instances),
- constrain the order of attribute evaluation,
- are input to attribute-evaluator generators.

Types of Dependencies

Local dependencies between attribute occurrences in a production according to a semantic rule,

Individual dependency graph of attribute instances of a tree obtained by pasting together local dependency graphs of productions (instances)

Global dependencies between attributes of a non-terminal induced by individual dependency graphs.

- An individual dependency graph may contain a cycle. Attribute instances on this cycle can not be evaluated.
- AG is **noncircular** if none of its individual dependency graphs contains a cycle.

Theorem

AG is well–formed iff it is noncircular.

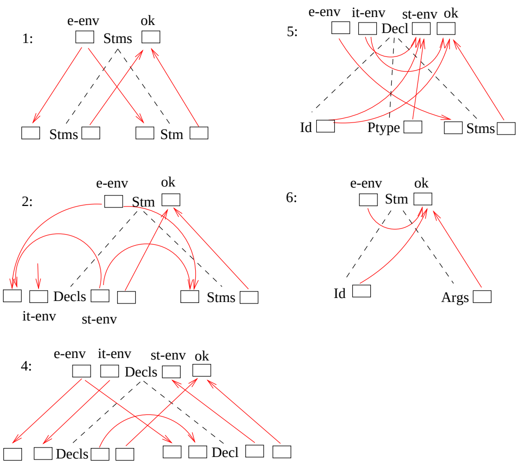
Local Dependencies

- **production local dependency relation** $Dp(p) \subseteq O(p) \times O(p)$:

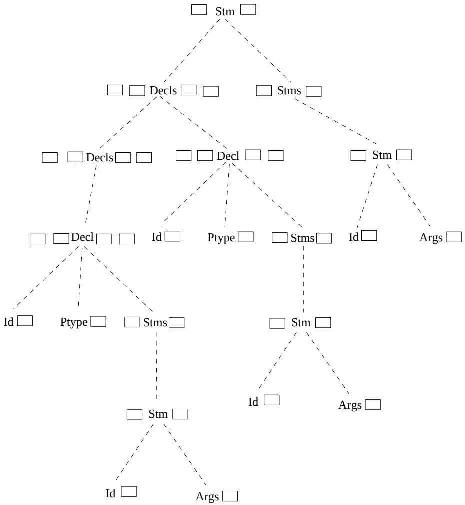
$$b_j \; Dp(p) \; a_i \qquad \text{iff} \qquad a_i = f_{p,a,i}(\ldots, b_j, \ldots)$$

- Attribute occurrence a_i at X_i depends on b_j at X_j iff b_j is argument in the semantic rule of a_i.
- Representation of this relation by its directed graph, the **production local dependency graph**, also denoted by $Dp(p)$.

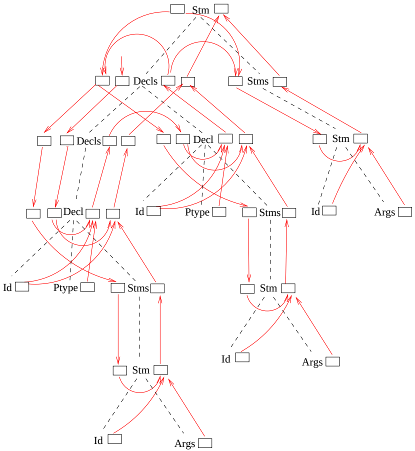
Local Dependencies in the Scopes-AG



Individual Dependency Graph



Individual Dependency Graphs



A First Attribute Evaluator

Principle:

1. Topological sorting of the individual dependency graph of a tree.
2. Attribute evaluation then done in the resulting order.

Topological sorting

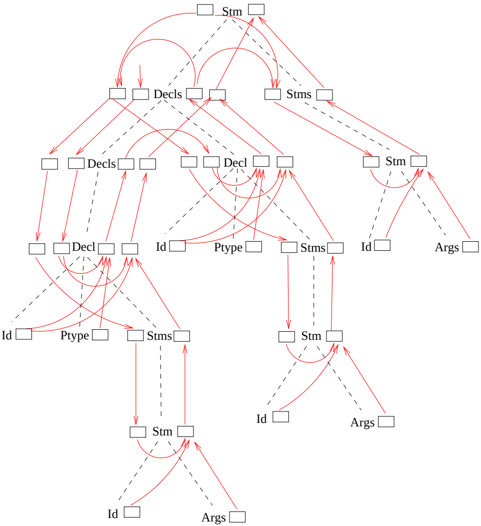
- takes a partial order (an acyclic graph),
- produces a total order compatible with the partial order,
- i.e., resulting total order, an **evaluation order**.

Topological sorting

- Keeps a set of **candidates** to be inserted next into the total order,
  initialized with the minimal elements of the order,
- In each step
  - Selects a candidate and inserts it into the total order,
  - Removes it from the set of candidates,
  - Removes it from the partial order,
  - Makes all elements only depending on this candidate to candidates,
- Until the set of candidates is empty.
- Partial order nonempty $\Rightarrow$ graph acyclic.

Can serve as a *dynamic* test for well formedness.

Example Evaluation



Properties of this Evaluator

- Evaluation order determined at evaluation time, i.e. compile time; therefore this evaluator is called the **dynamic** evaluator,
- Additional effort for the determination of the evaluation order at evaluation time,
- "Data driven" strategy, i.e. the availability of its arguments triggers the evaluation of an instance,
- Evaluates all instances in a tree,
- Evaluates each instance exactly once.

Alternatives

- Evaluation order may be fixed before evaluation time, e.g. by a fixed evaluation “plan” for each production,
- “Demand driven” strategy
  - Starts with a demand of some *maximal* elements in the partial order,
  - Demand for evaluation is passed to arguments,
  - Computed values are passed back.
- Properties of the demand driven strategy:
  - Allows the selective evaluation of a subset of “interesting” attribute instances,
  - Only instances needed for the evaluation of these attribute instances are evaluated,
  - May evaluate instances several times, depending on the implementation, i.e. on whether computed values are stored.

Issues

- Separation into

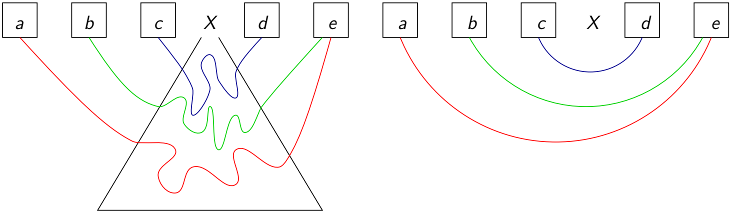
  Strategy phase: Evaluation order is determined,
  Evaluation phase: Evaluation proper of the attribute instances directed by this evaluation strategy.
- Goal: Preparation of the strategy phase at generation time, i.e., evaluation orders, evaluation plans, etc. are precomputed from the AG;
  may include a *static* test for well formedness,
- Complexity of

  Generation: Runtime in terms of AG size,
  Evaluation: Size of evaluator, time optimality of evaluation.
- AG subclasses, hierarchy: Expressivity, Generation algorithms, Complexity.

Lower Characteristic Graphs

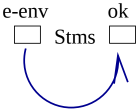
Given t, tree with root label X

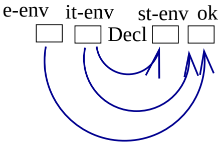
- "Projecting" the dependencies in $Dt(t)$ onto the attributes of X yields the **lower characteristic graph of X induced by t**, $Dt{\uparrow}_t(X)$.
- $Dt{\uparrow}_t(X)$ contains an edge from $a \in Inh(X)$ to $b \in Syn(X)$ iff there exists a path from the instance of a at the root to the instance of b at the root in $Dt(t)$.

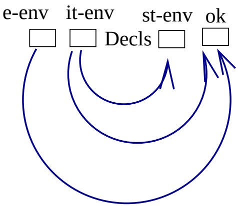


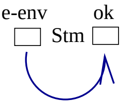
Example: Lower Characteristic Graphs

Lower characteristic graphs induced by the previous individual dependency graph:





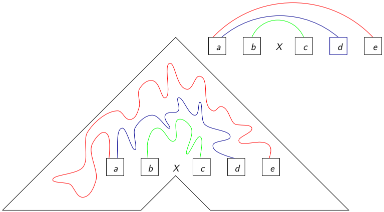




Upper Characteristic Graphs

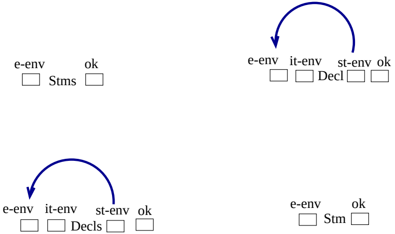
n inner node in t labeled X,
regards the upper tree fragment of t at n, $t\backslash n$,

- "Projecting" the dependencies in $Dt(t\backslash n)$ onto the attributes of X yields the **upper characteristic graph of X induced by** t, $Dt{\downarrow}_{t,n}(X)$.
- $Dt{\downarrow}_{t,n}(X)$ contains an edge from $a \in Syn(X)$ to $b \in Inh(X)$ iff there exists a path from the instance of a at n to the instance of b at n in $Dt(t\backslash n)$.

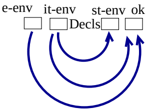


Example: Upper Characteristic Graphs

Upper characteristic graphs induced by the previous individual dependency graph:



Strategic Information in Characteristic Graphs

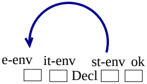


at the root of a subtree means:

it-env evaluated ⇒ st-env can be evaluated

e-env not evaluated ⇒ ok cannot be evaluated

during a downward visit.



at the root of a subtree means:

st-env unevaluated ⇒ e-env cannot be evaluated during an upward visit;

Induced Global Dependencies

The induction of characteristic graphs:

1. Local dependency graphs, $Dp(p)$: Relation on attribute **occurrences** of p
   Type conversion + Pasting
2. Individual dependency graph, $Dt(t)$: Relation on attribute **instances** in t
   Transitive closure and restriction
3. Relation on attribute **instances** of node n with $sym(n) = X$
   Type conversion
4. Lower characteristic graph $Dt{\uparrow}_t(X) \subseteq Inh(X) \times Syn(X)$: Relation on **attributes** of X or
5. Upper characteristic graph $Dt{\downarrow}_{t,n}(X) \subseteq Syn(X) \times Inh(X)$: Relation on **attributes** of X.

Computation of Global Dependency Graphs

- So far, **the** characteristic graph induced by **one** tree (fragment).
- Nonterminal X has
  - a set, $Dt{\uparrow}(X)$, of lower characteristic graphs and
  - a set, $Dt{\downarrow}(X)$, of upper characteristic graphs.
- These sets are computed at **generation time** by GFA.
- Only non–terminals can contribute, i.e., for $p : X_0 \to X_1 \dots X_{n_p}$ this means $X_i \in V_N$ for all $1 \leq i \leq n_p$..
- Watch out for "typing problems"!

Formalization of “Pasting”

$R_0, R_1, \ldots, R_{n_p}$ relations on the sets
$Attr(X_0), Attr(X_1), \ldots, Attr(X_{n_p})$, resp.
The pasting operation $Dp(p)[\cdot]$ has functionality
$Attr(X_0)^2 \times Attr(X_1)^2 \times \ldots \times Attr(X_{n_p})^2 \rightarrow O(p) \times O(p)$.
$Dp(p)[R_0, R_1, \ldots, R_{n_p}]$ is the following relation on $O(p)$:

$$Dp(p) \cup R_0^0 \cup R_1^1 \cup \ldots \cup R_{n_p}^{n_p},$$

where $b_i \; R_i^i \; a_i$ iff $b \; R_i \; a$.
The relations on the attributes of $X_0, X_1, \ldots, X_{n_p}$ are regarded as relations on attribute occurrences and unioned.
We write $Dp(p)[\emptyset, R_1, \ldots, R_{n_p}]$ as $Dp(p)[R_1, \ldots, R_{n_p}]$.

Formalization of Upward "Projection"

Upward projection $R{\uparrow}(p)[\cdot]$ has functionality:
$Attr(X_1)^2 \times \ldots \times Attr(X_{n_p})^2 \rightarrow Inh(X_0) \times Syn(X_0)$.
$R{\uparrow}(p)[R_1, \ldots, R_n]$ is the following relation:

$$b \; R{\uparrow}(p)[R_1, \ldots, R_n] \; a \text{ iff } b_0 \; Dp(p)[R_1, \ldots, R_n]^+ \; a_0.$$

Formalization of Downward "Projection"

Downward projection $R{\downarrow_i}(p)[\cdot]$ has functionality:
$Attr(X_0)^2 \times Attr(X_1)^2 \times \ldots \times Attr(X_{n_p})^2 \rightarrow Syn(X_i) \times Inh(X_i)$
$R{\downarrow_i}(p)[R_0, R_1, \ldots, R_{n_p}]$ is defined by

$$b \; R{\downarrow_i}(p)[R_0, R_1, \ldots, R_{n_p}] \; a \qquad\qquad \text{iff}$$

$$b_i \;\; Dp(p)[R_0, R_1, \ldots, R_{i-1}, \emptyset, R_{i+1}, \ldots, R_{n_p}]^+ \;\; a_i$$

Extensions to Sets of Relations

Let $\mathcal{R}_1, \ldots, \mathcal{R}_{n_p}$ be sets of relations on $Attr(X_1), \ldots, Attr(X_{n_p})$, resp.

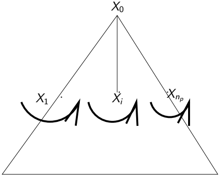
$$
\begin{array}{rcl}
R{\uparrow}(p)[\mathcal{R}_1, \ldots, \mathcal{R}_{n_p}] & = & \{R{\uparrow}(p)[R_1, \ldots, R_{n_p}] \mid R_i \in \mathcal{R}_i, (1 \leq i \leq n_p)\} \text{ and} \\
R{\downarrow}_i(p)[\mathcal{R}_0, \mathcal{R}_1, \ldots, \mathcal{R}_{n_p}] & = & \{R{\downarrow}_i(p)[R_0, R_1, \ldots, R_{n_p}] \mid R_j \in \mathcal{R}_j \ (0 \leq j \leq n_p)\} \\
 & & \text{for all } i \text{ in } (1 \leq i \leq n_p).
\end{array}
$$

GFA: Lower Characteristic Graphs

Evaluation time:

How to compute $Dt{\uparrow}_t(X_0)$ for a tree t with root label X_0 and $prod(\varepsilon) = p : X_0 \to X_1 \ldots X_{n_p}$?

Let the relations $Dt{\uparrow}_{t/1}(X_1), \ldots, Dt{\uparrow}_{t/n_p}(X_{n_p})$ be already computed.



Compute $Dt{\uparrow}_t(X_0)$ locally as

$$Dt{\uparrow}_t(X_0) = R{\uparrow}(p)[Dt{\uparrow}_{t/1}(X_1), \ldots, Dt{\uparrow}_{t/n_p}(X_{n_p})]$$

GFA: Lower Characteristic Graphs cont'd

This suggests for the **generation time:**

$$Dt{\uparrow}(X) = \bigcup_{p\,:\,p[0]\,=\,X} R{\uparrow}(p)[Dt{\uparrow}(p[1]), \ldots, Dt{\uparrow}(p[n_p])]$$

Least fixpoint is the set of the sets of lower characteristic graphs.

GFA–Problem Lower Characteristic Graphs

One step in the fixpoint iteration for production p:

1. Paste all combinations of lower characteristic graphs onto $Dp(p)$,
2. Project the resulting graphs onto the attributes of X_0,
3. Form the union all the resulting sets for X_0.

bottom up-GFA-Problem lower characteristic graphs	
lattices	$\{D(X) = \mathcal{P}(\mathcal{P}(Inh(X) \times Syn(X)))\}_{X \in V_N}$
part. order	$\subseteq$ (subset inclusion on sets of relations)
bottom	$\emptyset$ (empty set of relations)
transf. fct.	$\{Lc_p : D(p[1]) \times \ldots \times D(p[n_p]) \to D(p[0]) \mid$ $Lc_p(\mathcal{R}_1, \ldots, \mathcal{R}_{n_p}) = R{\uparrow}(p)[\mathcal{R}_1, \ldots, \mathcal{R}_{n_p}]\,\}_{p \in \mathcal{P}}$
comb. fct.	$\cup$ (union on sets of relations)

A Static Non–circularity Test

- A lower char. graph represents all dependencies in the trees inducing it.
- Pasting all combinations of lower char. graphs onto local dep. graphs produces a cyclic graph if AG is circular.
  Hence:
- AG is noncircular iff
  all graphs in $Dp(p)[Dt{\uparrow}(X_1), \ldots, Dt{\uparrow}(X_{n_p})]$ for all productions p are noncyclic.
- $|\bigcup_X Dt{\uparrow}(X)|$ exponential in $|Attr|$.
- The non–circularity test is exponential.

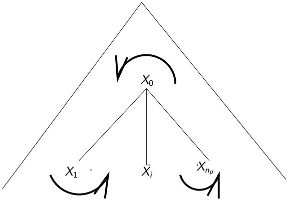
GFA: Upper Characteristic Graphs

Compile time:

Regard p applied at node n in t.

Already computed

$Dt{\downarrow}_{t,n}(X_0)$ and $Dt{\uparrow}_{t/n1}(X_1), \ldots, Dt{\uparrow}_{t/nn_p}(X_{n_p})$.



Compute $Dt{\downarrow}_{t,ni}(X_i)$ $(1 \leq i \leq n_p)$ using the operation $R{\downarrow}_i(p)[\ldots]$.

GFA: Upper Characteristic Graphs cont'd

This suggests for **generation time:**

$Dt{\downarrow}(S) = \{\emptyset\}$

$Dt{\downarrow}(X) = \bigcup_{p[i]\,=\,X} R{\downarrow}_i(p)[Dt{\downarrow}(p[0]), Dt{\uparrow}(p[1]), \ldots, Dt{\uparrow}(p[n_p])]$

Least fixpoint is the set of the sets of upper characteristic graphs.

GFA–Problem Upper Characteristic Graphs

top down-GFA-problem upper characteristic graphs	
Lattices	$\{D(X) = \mathcal{P}(\mathcal{P}(Syn(X) \times Inh(X)))\}_{X \in V_N}$
part. order	$\subseteq$ (subset inclusion on sets of relations)
bottom	$\emptyset$
transf. fct.	$\{Uc_{p,i} : D(p[0]) \rightarrow D(p[i])$
	$Uc_{p,i}(\mathcal{R}) = R{\downarrow_i}(p)[\mathcal{R}, Dt{\uparrow}(p[1]), \ldots, Dt{\uparrow}(p[n_p])]\,\}_{p \in \mathcal{P}, 1 \leq i \leq n_p}$
comb. fct.	$\cup$ (union on sets of relations)

- The sets of lower characteristic graphs are assumed to be computed before.
- They are constant parts of the functions $Uc_{p,i}$.

Resumee Characteristic Graphs

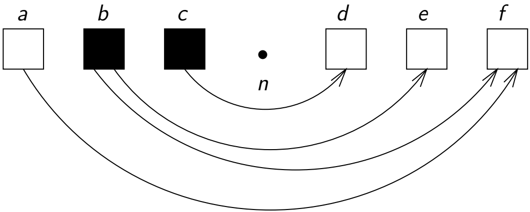
Characteristic graphs are:

Exact: For each characteristic graph there is at least one tree (fragment), whose individual dependency graph induces it,

Costly: There may be exponentially many of them.

Approximative Attribute Dependencies

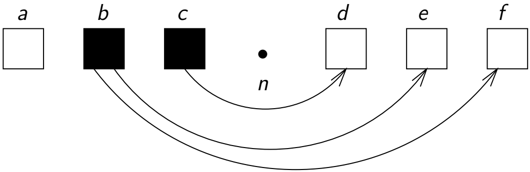
What is the "strategic" interpretation of edges in (lower) characteristic graphs?



Evaluator visits subtree at n with b, c evaluated.
Through this visit, it can

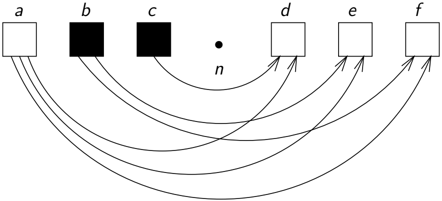
- evaluate d and e,
- not evaluate f.

What does "approximation" mean? deleting edges? adding edges?
Deleting the edge from a to f:



- Evaluator assumes, f can be evaluated when value of b is known.
- Makes a fruitless visit to the subtree at n.
- Inefficient strategy!

Adding edges from *a* to *d* and *e*:



- Evaluator would not visit the subtree at *n* with evaluated *b* and *c* and unevaluated *a*,
- Evaluator would only visit the subtree, when also the value of *a* is known.
- Visits may be delayed.

Resumee:

- Reduced dependency graphs may cause fruitless visits,
- Augmented dependency graphs may delay visits,
- Added edges may introduce cycles (cause an infinite delay).

I/O–Graphs

- Are an upper bound on the lower dependencies,
- There may be I/O–graphs with no corresponding tree,
- There is one graph per nonterminal.

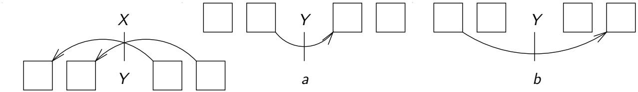
bottom up-GFA-problem I/O-graphs	
lattices	$\{D(X) = \mathcal{P}(Inh(X) \times Syn(X))\}_{X \in V_N}$
part. order	$\subseteq$ (subset inclusion on relations)
bottom	$\emptyset$
transf. fct.	$\{Io : D(p[1]) \times \ldots \times D(p[n_p]) \rightarrow D(p[0]) \mid$ $Io(g_1, \ldots, g_{n_p}) = R{\uparrow}(p)[g_1, \ldots, g_{n_p}]\}_{p \in \mathcal{P}}$
comb. fct.	$\cup$ (union on relations)

Yields the system of equations:

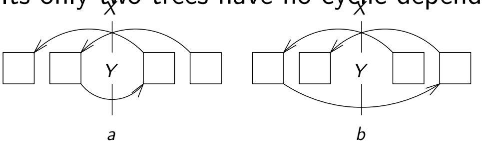
$$IO(X) = \bigcup_{p\,:\,p[0]\,=\,X} R{\uparrow}(p)[IO(p[1]), \ldots, IO(p[n_p])]$$

AG is **absolutely noncircular** if for all productions p the graph $Dp(p)[IO(p[1]), \ldots, IO(p[n_p])]$ is acyclic.

A Noncircular, but not Absolutely Noncircular AG



Its only two trees have no cyclic dependencies.



For computing $IO(X)$ $Dp(2)$ and $Dp(3)$ are unioned and inserted in $Dp(1)$ producing a cycle.