Syntax Analysis

Top-down Syntax Analysis

– Wilhelm/Seidl/Hack: Compiler Design, Syntactic and Semantic Analysis–

Reinhard Wilhelm
Universität des Saarlandes
wilhelm@cs.uni-saarland.de
and
Mooly Sagiv
Tel Aviv University
sagiv@math.tau.ac.il

Topics

- Functionality and Method
- Recursive Descent Parsing
- Using parsing tables
- Explicit stacks
- Creating the table
- LL(k)–grammars
- Other properties
- Handling Limitations

Top-Down Syntax Analysis

input: A sequence of symbols (tokens)

output: A syntax tree or an error message

method
- Read input from left to right
- Construct the syntax tree in a top-down manner starting with a node labeled with the start symbol
- **until** input accepted (or error) **do**
  - Predict expansion for the actual leftmost nonterminal (maybe using some lookahead into the remaining input) or
  - Verify predicted terminal symbol against next symbol of the remaining input

Finds leftmost derivations.

Grammar for Arithmetic Expressions

Left factored grammar G_2, i.e. left recursion removed.

$S \rightarrow E$	
$E \rightarrow TE'$	E generates T with a continuation E'
$E' \rightarrow +E \mid \epsilon$	E' generates possibly empty sequence of $+T$s
$T \rightarrow FT'$	T generates F with a continuation T'
$T' \rightarrow *T \mid \epsilon$	T' generates possibly empty sequence of $*F$s
$F \rightarrow \mathbf{id} \mid (E)$	

Recursive Descent Parsing

- parser is a program,
- a procedure X for each non-terminal X,
  - parses words for non-terminal X,
  - starts with the first symbol read (into variable *nextsym*),
  - ends with the following symbol read (into variable *nextsym*).
- uses one symbol lookahead into the remaining input.
- uses the **FiFo** sets to make the expansion transitions deterministic

$$\textbf{FiFo}(N \to \alpha) = FIRST_1(\alpha) \oplus_1 FOLLOW_1(N) = \begin{cases} FIRST_1(\alpha) \cup FOLLOW_1(N) & \alpha \stackrel{*}{\Longrightarrow} \epsilon \\ FIRST_1(\alpha) & \text{otherwise} \end{cases}$$

Parser for G_2

```
program parser;
var  nextsym: string;
proc  scan;
 {reads next input symbol into  nextsym}
proc  error (message: string);
 {issues error message and stops parser}
proc  accept; {terminates successfully}

proc  S;
 begin  E
 end ;

proc  E;
 begin T; E'
 end ;
```

```
proc  E';
 begin
   case  nextsym in
    {"+"}:  if  nextsym = "+ "
            then  scan
            else  error( "+ expected") fi ; E;
    otherwise ;
   endcase
 end ;

proc  T;
 begin F; T' end ;
proc  T';
 begin
   case  nextsym in
    {" * "}:  if  nextsym = "*"
             then  scan
             else  error( "* expected") fi ; T;
    otherwise ;
   endcase
 end ;
```

```
proc F;
 begin
   case nextsym in
    {"("}:    if nextsym = "("
              then scan
              else error( "( expected") fi ; E;
              if nextsym = ")"
              then scan else error(" ) expected") fi;
    otherwise if nextsym ="id"
              then scan else error("id expected") fi;
   endcase
 end ;
begin
scan; S;
if nextsym = "#" then accept else error("# expected") fi
end .
```

How to Construct such a Parser Program

Observation: Much redundant code generated. Why this?
Code was automatically generated from the grammar and the **FiFo** sets.
Nice application for a functional programming language!
Let $G = (V_N, V_T, P, S)$ be a context-free grammar and FiFo be the computed lookahead sets.
The functional program generating the parser would have the functions:

N_prog	:	$V_N \rightarrow$ code	nonterminals
C_prog	:	$(V_N \cup V_T)^* \rightarrow$ code	concantenations
S_prog	:	$V_N \cup V_T \rightarrow$ code	symbols

Parser Schema

```
program parser;
    var  nextsym: symbol;
    proc  scan;
        (* reads next input symbol into  nextsym *)
    proc  error (message: string);
        (* issues error message and stops the parser *)
    proc  accept;
        (* terminates parser successfully *)

    N_prog(X0);                                  (* X0 start symbol *)
    N_prog(X1);
        ⋮
    N_prog(Xn);
```

```
begin
    scan;
    X0;
    if  nextsym = "#"
        then  accept
        else  error("...")
    fi
end
```

The Non-terminal Procedures

N = Non-terminal, C = Concatenation, S = Symbol

N_prog(X) = (* $X \to \alpha_1|\alpha_2|\cdots|\alpha_{k-1}|\alpha_k$ *)

proc X;
begin
case nextsym **in**
FiFo($X \to \alpha_1$) : C_progr(α_1);
FiFo($X \to \alpha_2$) : C_progr(α_2);
⋮
FiFo($X \to \alpha_{k-1}$) : C_progr(α_{k-1});
otherwise C_progr(α_k);
endcase
end ;

C_progr($\alpha_1 \alpha_2 \cdots \alpha_k$) =
S_progr(α_1); S_progr(α_2); ... S_progr(α_k);
S_progr(a) =
if nextsym = a **then** scan
else error ("a expected")
fi
S_progr(Y) = Y

FiFo–sets should be disjoint (LL(1)–grammar)

A Generative Solution

Generate the control of a deterministic PDA from the grammar and the **FiFo** sets.

- At compiler–generation time construct a table M
  $M : V_N \times V_T \rightarrow P$
  $M[N, a]$ is the production used to expand nonterminal N when the current symbol is a
- For some grammars report that the table cannot be constructed
  The compiler writer can then decide to:
  - change the grammar (but not the language)
  - use a more general parser-generator
  - "Patch" the table (manually or using some rules)

Creating the table

Input: cfg G, $FIRST_1$ und $FOLLOW_1$ for G.

Output: The parsing table M or an indication that such a table cannot be constructed

Method: M is constructed as follows:
For all $X \to \alpha \in P$ and $a \in FIRST_1(\alpha)$, set $M[X, a] = (X \to \alpha)$.
If $\varepsilon \in FIRST_1(\alpha)$, for all $b \in FOLLOW_1(X)$, set $M[X, b] = (X \to \alpha)$.
Set all other entries of M to *error* .

Parser table cannot be constructed if at least one entry is set twice.
G is not LL(1)

Example – arithmetic expressions

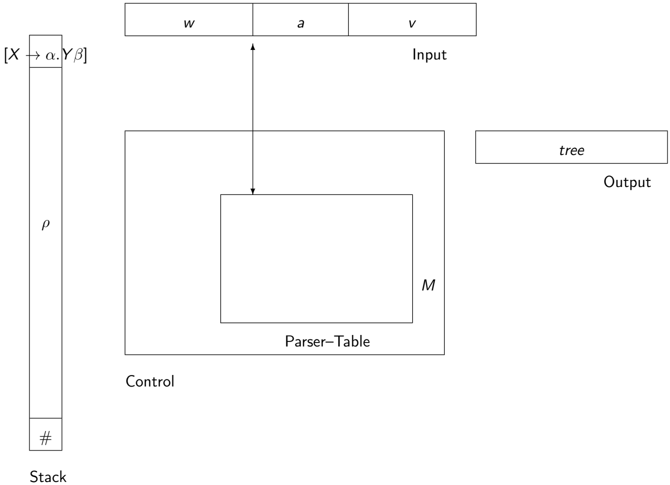
nonterminal	symbol	Production
S	$(, id$	$S \rightarrow E$
S	$+, *,), \#$	*error*
E	$(, id$	$E \rightarrow TE'$
E	$+, *,), \#$	*error*
E'	$+$	$E' \rightarrow +E$
E'	$), \#$	$E' \rightarrow \epsilon$
E'	$(, *, id$	*error*
T	$(, id$	$T \rightarrow FT'$
T	$+, *,), \#$	*error*
T'	$*$	$T' \rightarrow *T$
T'	$+,), \#$	$T' \rightarrow \epsilon$
T'	$(, id$	*error*
F	id	$F \rightarrow id$
F	$($	$F \rightarrow (E)$
F	$+, *,)$	*error*

LL-Parser Driver (interprets the table M)

```
program parser;
    var  nextsym: symbol;
    var  st: stack of  item;
    proc  scan;
        (* reads next input symbol into  nextsym *)
    proc  error (message: string);
        (* issues error message and stops the parser *)
    proc  accept;
        (* terminates parser successfully *)
    proc reduce;
        (* replaces [X → β.Yγ][Y → α.] by [X → βY.γ]  *)
    proc pop;
        (* removes topmost item from st *)
    proc push ( i : item);
        (* pushes i onto st *)
    proc replaceby ( i: item);
        (* replaces topmost item of st by i *)
```

```
begin
    scan; push( [S' → .S] );
    while  nextsym ≠ "#" do
    case top in
    [X → β.aγ]:      if  nextsym = a
                     then  scan; replaceby([X → βa.γ]
                     else error fi ;
    [X → β.Yγ] :     if  M[Y, nextsym] = (Y → α)
                     then  push([Y → .α])
                     else  error fi ;
    [X → α.]:        reduce;
    [S' → S.] :      if  nextsym = "#" then  accept
                     else  error fi
    endcase
    od
end  .
```

Explicit Stack
Deterministic Pushdown Automaton



LL(k)-grammar

Goal: formalizing our intuition when the expand-transitions of the Item-Pushdown-Automaton can be made deterministic.

Means: k-symbol lookahead into the remaining input.

LL(k)-grammar

Let $G = (V_N, V_T, P, S)$ be a cfg and k be a natural number.
G is an **LL(k)-grammar** iff the following holds:
if there exist two leftmost derivations

$$S \underset{lm}{\overset{*}{\Longrightarrow}} uY\alpha \underset{lm}{\Longrightarrow} u\beta\alpha \underset{lm}{\overset{*}{\Longrightarrow}} ux \text{ and}$$

$$S \underset{lm}{\overset{*}{\Longrightarrow}} uY\alpha \underset{lm}{\Longrightarrow} u\gamma\alpha \underset{lm}{\overset{*}{\Longrightarrow}} uy, \text{ and if } k : x = k : y,$$

then $\beta = \gamma$.

The expansion of the leftmost non-terminal is always uniquely determined by

- the consumed part of the input and
- the next k symbols of the remaining input

LL(k)-grammar

Let $G = (V_N, V_T, P, S)$ be a cfg and k be a natural number.
G is an **LL(k)-grammar** iff the following holds:
if there exist two leftmost derivations

$$S \overset{*}{\underset{lm}{\Longrightarrow}} uY\alpha \underset{lm}{\Longrightarrow} u\beta\alpha \overset{*}{\underset{lm}{\Longrightarrow}} ux \text{ and}$$

$$S \overset{*}{\underset{lm}{\Longrightarrow}} uY\alpha \underset{lm}{\Longrightarrow} u\gamma\alpha \overset{*}{\underset{lm}{\Longrightarrow}} uy, \text{ and if } k : x = k : y,$$

then $\beta = \gamma$.
The expansion of the leftmost non-terminal is always uniquely determined by

- the consumed part of the input and
- the next k symbols of the remaining input

Example 1

Let G_1 be the cfg with the productions

$$
\begin{array}{lll}
STAT & \rightarrow & \textbf{if}\ \ \textbf{id then}\ STAT\ \textbf{else}\ STAT\ \textbf{fi} \quad | \\
 & & \textbf{while}\ \ \textbf{id do}\ STAT\ \textbf{od} \quad | \\
 & & \textbf{begin}\ STAT\ \textbf{end} \quad | \\
 & & \textbf{id} := \textbf{id}
\end{array}
$$

G_1 is an LL(1)-grammar.

$$STAT \overset{*}{\underset{lm}{\Longrightarrow}} w\ STAT\ \alpha \underset{lm}{\Longrightarrow} w\ \beta\ \alpha \overset{*}{\underset{lm}{\Longrightarrow}} w\ x$$

$$STAT \overset{*}{\underset{lm}{\Longrightarrow}} w\ STAT\ \alpha \underset{lm}{\Longrightarrow} w\ \gamma\ \alpha \overset{*}{\underset{lm}{\Longrightarrow}} w\ y$$

From $1 : x = 1 : y$ follows $\beta = \gamma$,
e.g., from $1 : x = 1 : y =$ **if** follows
$\beta = \gamma =$ "**if id then** $STAT$ **else** $STAT$ **fi**"

Example 1

Let G_1 be the cfg with the productions

$$
\begin{array}{lll}
STAT & \rightarrow & \textbf{if id then } STAT \textbf{ else } STAT \textbf{ fi} \quad | \\
& & \textbf{while id do } STAT \textbf{ od} \quad | \\
& & \textbf{begin } STAT \textbf{ end} \quad | \\
& & \textbf{id} := \textbf{id}
\end{array}
$$

G_1 is an LL(1)-grammar.

$$
\begin{array}{lllllll}
STAT & \overset{*}{\underset{lm}{\Longrightarrow}} & w\ STAT\ \alpha & \underset{lm}{\Longrightarrow} & w\,\beta\,\alpha & \overset{*}{\underset{lm}{\Longrightarrow}} & w\,x \\
STAT & \overset{*}{\underset{lm}{\Longrightarrow}} & w\ STAT\ \alpha & \underset{lm}{\Longrightarrow} & w\,\gamma\,\alpha & \overset{*}{\underset{lm}{\Longrightarrow}} & w\,y
\end{array}
$$

From $1 : x = 1 : y$ follows $\beta = \gamma$,
e.g., from $1 : x = 1 : y = \textbf{if}$ follows
$\beta = \gamma =$ "**if id then** $STAT$ **else** $STAT$ **fi**"

Example 2

Let G_2 be the cfg with the productions

$$
\begin{array}{rcll}
STAT & \rightarrow & \textbf{if}\ \ \textbf{id then}\ STAT\ \textbf{else}\ STAT\ \textbf{fi} & | \\
 & & \textbf{while}\ \ \textbf{id do}\ STAT\ \textbf{od} & | \\
 & & \textbf{begin}\ STAT\ \textbf{end} & | \\
 & & \textbf{id} := \textbf{id}\ \ | & \\
 & & \textbf{id}{:}\ STAT\ \ | & (*\ \text{labeled statem.}\ *) \\
 & & \textbf{id}\,(\textbf{id}\,) & (*\ \text{procedure call}\ *)
\end{array}
$$

Example 2 (cont'd)

G_2 is not an LL(1)–grammar.

$$STAT \underset{lm}{\overset{*}{\Longrightarrow}} w\ STAT\ \alpha \underset{lm}{\Longrightarrow} w\ \overbrace{\mathbf{id} := \mathbf{id}}^{\beta}\ \alpha \underset{lm}{\overset{*}{\Longrightarrow}} w\ x$$

$$STAT \underset{lm}{\overset{*}{\Longrightarrow}} w\ STAT\ \alpha \underset{lm}{\Longrightarrow} w\ \overbrace{\mathbf{id} : STAT}^{\gamma}\ \alpha \underset{lm}{\overset{*}{\Longrightarrow}} w\ y$$

$$STAT \underset{lm}{\overset{*}{\Longrightarrow}} w\ STAT\ \alpha \underset{lm}{\Longrightarrow} w\ \overbrace{\mathbf{id}(\mathbf{id})}^{\delta}\ \alpha \underset{lm}{\overset{*}{\Longrightarrow}} w\ z$$

and $1 : x = 1 : y = 1 : z =$ "**id**",
and β, γ, δ are pairwise different.
G_2 is an LL(2)–grammar.
$2 : x =$ "**id** :=", $2 : y =$ "**id** :", $2 : z =$ "**id**(" are pairwise different.

Example 3

Let G_3 have the productions

$$
\begin{array}{lcll}
STAT & \rightarrow & \textbf{if}\ \ \textbf{id}\ \textbf{then}\ \ STAT\ \textbf{else}\ \ STAT\ \textbf{fi} & | \\
 & & \textbf{while}\ \ \textbf{id}\ \textbf{do}\ STAT\ \textbf{od} & | \\
 & & \textbf{begin}\ \ STAT\ \textbf{end} & | \\
 & & VAR := VAR & | \\
 & & \textbf{id}(\ IDLIST\ ) & (*\ \text{procedure call}\ *) \\
VAR & \rightarrow & \textbf{id}\ \ |\ \ \textbf{id}\ (IDLIST\ ) & (*\ \text{indexed variable}\ *) \\
IDLIST & \rightarrow & \textbf{id}\ \ |\ \ \textbf{id},\ IDLIST &
\end{array}
$$

G_3 is not an LL(k)–grammar for any k.

Example 3

Let G_3 have the productions

$$
\begin{array}{lcll}
STAT & \rightarrow & \textbf{if}\ \ \textbf{id}\ \textbf{then}\ \ STAT\ \textbf{else}\ \ STAT\ \textbf{fi} & | \\
 & & \textbf{while}\ \ \textbf{id}\ \textbf{do}\ STAT\ \textbf{od} & | \\
 & & \textbf{begin}\ \ STAT\ \textbf{end} & | \\
 & & VAR := VAR & | \\
 & & \textbf{id}(\ IDLIST\ ) & (*\ \text{procedure call}\ *) \\
VAR & \rightarrow & \textbf{id}\ \mid\ \textbf{id}\,(IDLIST\,) & (*\ \text{indexed variable}\ *) \\
IDLIST & \rightarrow & \textbf{id}\ \mid\ \textbf{id},\ IDLIST &
\end{array}
$$

G_3 is not an LL(k)–grammar for any k.

Proof:

Assume G_3 to be LL(k) for a $k > 0$.

Let $STAT \Rightarrow \beta \overset{*}{\underset{lm}{\Longrightarrow}} x$ and $STAT \Rightarrow \gamma \overset{*}{\underset{lm}{\Longrightarrow}} y$ with

$$x = \textbf{id}\,(\underbrace{\textbf{id}, \textbf{id}, \ldots, \textbf{id}}_{\lceil \frac{k}{2} \rceil \text{ times}}) := \textbf{id} \text{ and } y = \textbf{id}\,(\underbrace{\textbf{id}, \textbf{id}, \ldots, \textbf{id}}_{\lceil \frac{k}{2} \rceil \text{ times}})$$

Then $k : x = k : y$,
but $\beta = "VAR := VAR" \neq \gamma = "\textbf{id}\,(IDLIST)"$.

Transforming to LL(k)

Factorization creates an LL(2)–grammar, equivalent to G_3.

The productions

$$STAT \rightarrow VAR := VAR \mid \mathbf{id}(IDLIST)$$

are replaced by

$$\begin{array}{ll} STAT & \rightarrow ASSPROC \mid \mathbf{id} := VAR \\ ASSPROC & \rightarrow \mathbf{id}(IDLIST)\ APREST \\ APREST & \rightarrow := VAR \mid \varepsilon \end{array}$$

A non–LL(k)–language

Let $G_4 = (\{S, A, B\}, \{0, 1, a, b\}, P_4, S)$

$$P_4 = \left\{ \begin{array}{lcl} S & \rightarrow & A \mid B \\ A & \rightarrow & aAb \mid 0 \\ B & \rightarrow & aBbb \mid 1 \end{array} \right\}$$

$L(G_4) = \{a^n 0 b^n \mid n \geq 0\} \cup \{a^n 1 b^{2n} \mid n \geq 0\}$.

G_4 is not LL(k) for any k.

Consider the two leftmost derivations

$$S \overset{0}{\underset{lm}{\Longrightarrow}} S \underset{lm}{\Longrightarrow} A \overset{*}{\underset{lm}{\Longrightarrow}} a^k 0 b^k$$

$$S \overset{0}{\underset{lm}{\Longrightarrow}} S \underset{lm}{\Longrightarrow} B \overset{*}{\underset{lm}{\Longrightarrow}} a^k 1 b^{2k}$$

With $u = \alpha = \varepsilon$, $\beta = A$, $\gamma = B$, $x =$ "$a^k 0 b^k$", $y =$ "$a^k 1 b^{2k}$" it holds $k : x = k : y$, but $\beta \neq \gamma$.

Since k can be chosen arbitrarily, we have G_4 is not LL(k) for any k.

There even is no LL(k)-grammar for $L(G_4)$ for any k.

Towards Checkable LL(k)–conditions

Theorem

G is LL(k)–grammar iff the following condition holds:
Are $A \to \beta$ and $A \to \gamma$ different productions in P, then

$$FIRST_k(\beta\alpha) \cap FIRST_k(\gamma\alpha) = \emptyset \quad \text{for all } \alpha \quad \text{with } S \overset{*}{\underset{lm}{\Longrightarrow}} wA\alpha$$

Theorem

Let G be a cfg without productions of the form $X \to \varepsilon$.
G is an LL(1)–grammar iff
for each non-terminal X with the alternatives $X \to \alpha_1 | \ldots | \alpha_n$
the sets $FIRST_1(\alpha_1), \ldots, FIRST_1(\alpha_n)$ are pairwise disjoint.

Theorem

G is LL(1) iff
For different productions $A \rightarrow \beta$ and $A \rightarrow \gamma$
$FIRST_1(\beta) \oplus_1 FOLLOW_1(A) \cap FIRST_1(\gamma) \oplus_1 FOLLOW_1(A) = \emptyset$.

Corollary:

G is LL(1) iff for all alternatives $A \rightarrow \alpha_1 | \dots | \alpha_n$:

1. $FIRST_1(\alpha_1), \dots, FIRST_1(\alpha_n)$ are pairwise disjoint; in particular, at most one of them may contain ε
2. $\alpha_i \stackrel{*}{\Longrightarrow} \varepsilon$ implies:

   $$FIRST_1(\alpha_j) \cap FOLLOW_1(A) = \emptyset \text{ for } 1 \leq j \leq n,\ j \neq i.$$

The condition of the Theorem was used in the parser construction!

Further Definitions and Theorems

- G is called a **strong LL(k)-grammar** if for each two different productions $A \rightarrow \beta$ and $A \rightarrow \gamma$

$$FIRST_k(\beta) \oplus_k FOLLOW_k(A) \cap FIRST_k(\gamma) \oplus_k FOLLOW_k(A) = \emptyset,$$

- A production is called **directly left recursive**, if it has the form $A \rightarrow A\alpha$
- A non-terminal A is called **left recursive** if it has a derivation $A \stackrel{+}{\Longrightarrow} A\alpha$.
- A cfg G is called **left recursive**, if G contains at least one left recursive non-terminal

Theorem

(a) *G is not $LL(k)$ for any k if G is left recursive.*

(b) *G is not ambiguous if G is $LL(k)$-grammar.*

Recursive Descent Parsing

```
program parser;
    var nextsym: symbol;
    proc scan;
        (* reads next input symbol into nextsym *)
    proc error (message: string);
        (* issues error message and stops the parser *)
    proc accept;
        (* terminates parser successfully *)
    N_prog(X0 → α0);
    N_prog(X1 → α1);
        ⋮
    N_prog(Xn → αn); begin
    scan;
    X0;
    if nextsym = "#"
        then accept
        else error("...")
    fi
end
```

$N\_prog(X \rightarrow \alpha) =$

proc X;

begin

progr($[X \rightarrow .\alpha]$)

end ;

progr($[X \rightarrow \cdots .(\alpha_1|\alpha_2|\cdots|\alpha_{k-1}|\alpha_k)\cdots]$) =

case nextsym **in**

FiFo($[X \rightarrow \cdots(.\alpha_1|\alpha_2|\cdots|\alpha_{k-1}|\alpha_k)\cdots]$) :

progr($[X \rightarrow \cdots(.\alpha_1|\alpha_2|\cdots|\alpha_{k-1}|\alpha_k)\cdots]$);

FiFo($[X \rightarrow \cdots(\alpha_1|.\alpha_2|\cdots|\alpha_{k-1}|\alpha_k)\cdots]$) :

progr($[X \rightarrow \cdots(\alpha_1|.\alpha_2|\cdots|\alpha_{k-1}|\alpha_k)\cdots]$);

$\vdots$

FiFo($[X \rightarrow \cdots(\alpha_1|\alpha_2|\cdots|.\alpha_{k-1}|\alpha_k)\cdots]$) :

progr($[X \rightarrow \cdots(\alpha_1|\alpha_2|\cdots|.\alpha_{k-1}|\alpha_k)\cdots]$);

otherwise progr($[X \rightarrow \cdots(\alpha_1|\alpha_2|\cdots|\alpha_{k-1}|.\alpha_k)\cdots]$);

endcase

$$
\begin{aligned}
&\text{progr}([X \to \cdots .(\alpha_1\alpha_2\cdots\alpha_k)\cdots]) = \\
&\qquad \text{progr}([X \to \cdots(.\alpha_1\alpha_2\cdots\alpha_k)\cdots]); \\
&\qquad \text{progr}([X \to \cdots(\alpha_1.\alpha_2\cdots\alpha_k)\cdots]); \\
&\qquad\qquad\qquad \vdots \\
&\qquad \text{progr}([X \to \cdots(\alpha_1\alpha_2\cdots .\alpha_k)\cdots]); \\
&\text{progr}([X \to \cdots .(\alpha)^* \cdots]) = \\
&\qquad \textbf{while}\ \ \text{nextsym}\ \textbf{in}\ \ \mathit{FIRST}_1(\alpha)\ \textbf{do} \\
&\qquad\qquad \text{progr}([X \to \cdots .\alpha\cdots]) \\
&\qquad \textbf{od}
\end{aligned}
$$

For $a \in V_T$ is
$\text{progr}([X \rightarrow \cdots .a \cdots]) =$

if nextsym = a **then** scan
else error
fi

For $Y \in V_N$ is
$\text{progr}([X \rightarrow \cdots .Y \cdots]) = Y$