Exercise 7.1 SSA-Property

Which of the following control flow graphs represent valid SSA-form programs? Justify your answer.

Exercise 7.2 Static Single Assignment (SSA) Form and Sparse Conditional Constant Propagation (SCCP)

Consider the following program $P$:

```plaintext
x = 1;
y = 1;
while (...) {
    if (x != 1)
        y = 2;
    z = x;
x = 2;
    if (y == 1)
        x = z;
}
print(x);
```
1. Construct the abstract syntax tree (AST) from $P$.

2. Compute SSA form for $P$ by using Cytron’s algorithm:
   a) Construct the control flow graph (CFG) from $P$. Label each basic block with an uppercase latter.
   b) Construct the dominance tree for the CFG.
   c) Calculate the dominance frontier $DF$ and the iterated dominance frontier $DF^+$ for each basic block.
   d) Give each definition a unique name. Furthermore, create names for the result of a comparison.
   e) Insert $\phi$ functions and rewires all uses.

3. Compute SSA form for $P$ by using Braun et. al.’s simple algorithm:
   a) Traverse the AST by depth-first search (visit the children from left to right). This corresponds to traversing the program text from top to bottom.
   b) Successively create the CFG and fill it with instructions in SSA form. Note that you don’t create “copy-instructions” when using this algorithm. Remember to create names for the result of a comparison. Seal basic blocks as early as possible. Insert $\phi$ functions on the fly.

4. Why does Cytron’s algorithm insert more $\phi$ functions than Braun et. al.’s algorithm?

5. SCCP is a combination of three data-flow analyses: Constant Propagation Analysis, Reachability Analysis and Constant Branch Analysis. The SCCP lattice is therefor the cross product of the lattices of each individual analysis and the transformer can use information from all three domains. Answer the following questions and perform SCCP on $P$ in SSA form:
   a) Write down all lattices (reachability, general constants, boolean constants for conditions).
   b) Write down the transfer functions.
   c) Draw a table with one row for each SSA variable and one row for each basic block.
   d) Initialize all values in the first column.
   e) Now perform the algorithm till the fixed point. Record updates to a value in a new column. Perform the updates in a smart order to minimize work.
   f) Is SCCP less, equally or more powerful than the three individual analyses applied in any order and any number of repetitions? Justify your answer and give an example.

6. Effects of SSA form.
   a) Explain the difference between a flow-sensitive and a flow-insensitive analysis for both SSA and non-SSA form programs.
   b) Consider a program $P$ with $|V|$ variables and $|B|$ basic blocks ($O(|V|) = O(|B|) = O(|P|)$). Estimate (using $O$-notation) the memory consumption (the size of the necessary tables) by the classic (non-sparse) and SSA-based sparse conditional constant propagation analyses.

For the SSA estimation, let $|\phi|$ be the number of $\phi$ functions and $|def|$ be the number of assignments in $P$ and consider these two cases:
   i. Suppose $O(|\phi|) = O(|def|)$.
   ii. Presume there are the maximum number of $\phi$ functions.