

Exercise Sheet 4

Exercise 4.1 Properties of $LR_0(G)$

Consider again the canonical $LR(0)$ automaton that is constructed from the characteristic automaton of some context free grammar G via the subset construction (e.g. for the simple expression grammar as presented in the lecture). One can observe that for every state q of $LR_0(G)$ that is not the initial state of the automaton, the incoming transitions of q all are under the same symbol.

In formulas, for $LR_0(G) = (Q, \Sigma, \Delta, q_0, F)$:

$$\forall q, q_1, q_2 \in Q. \forall a, b \in \Sigma. (\Delta(q_1, a) = q \wedge \Delta(q_2, b) = q) \Rightarrow a = b$$

Explain why this property holds (you do not need to give a formal proof).

Exercise 4.2 Type Inference

Consider the following program written in the toy functional language given in the lecture:

```
let rec f = fun ys → fun xs →
  if xs = [] then
    ys
  else
    ((f ((head xs) :: ys)) (tail xs))
in (f [])
```

We extend the language with the following built-in functions operating on lists:

$$\begin{aligned} head &: \alpha \text{ list} \rightarrow \alpha \\ tail &: \alpha \text{ list} \rightarrow \alpha \text{ list} \end{aligned}$$

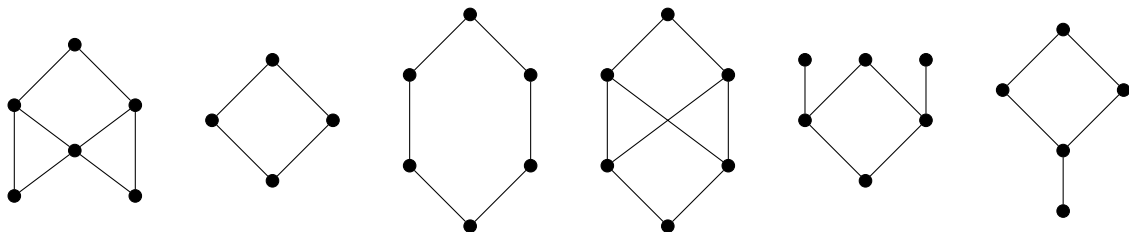
1. Construct the abstract syntax tree for the program.
2. Type-check the program by constructing the set of equations and computing a most general unifier with the algorithms presented in the lecture. What is the type of the top-level expression?
3. Type-check the program with the syntax-driven algorithm \mathcal{W} presented in the lecture.
4. What does the program compute?

Exercise 4.3 Soundness in the Rules-of-Signs Analysis

Give a program (i.e. a control flow graph) and a corresponding computation of the fix-point of the Rules-of-Signs abstraction as presented in the lecture such that an intermediate result (which is not yet a fix-point) is already a sound approximation of the concrete program semantics.

Exercise 4.4 Finite Lattices

1. Which of the partially ordered sets described by the following Hasse diagrams are lattices? Which of them are complete lattices?



2. Relate the set of all finite lattices with the set of all finite complete lattices. Prove your claims.