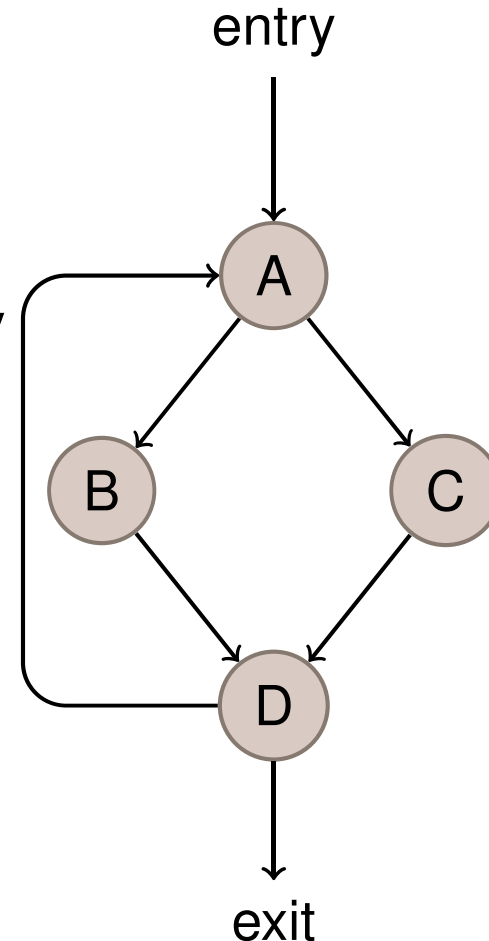


Notion of Persistence

- Intuition: “Block b is *persistent* if it can only cause one cache miss in any execution.”
- What is an appropriate concrete semantics that captures this property?
- Ideas for abstractions?

NEED “TRACE”
SEMANTICS



COLLECTING TRACE SEMANTICS

$$L \rightarrow P(\text{TRACES})$$

↑
SET OF PROGRAM
POINTS

CAUSE STATES

$$\text{TRACES} \stackrel{?}{=} C' \times \left[(M \times \{\text{lit}, \text{mem}\}) \times C \right]^*$$

$$c_1 - (m_1, \underline{\text{lit}}_1) \rightarrow c_2 - (m_2, \underline{\text{lit}}_2) \rightarrow c_3$$

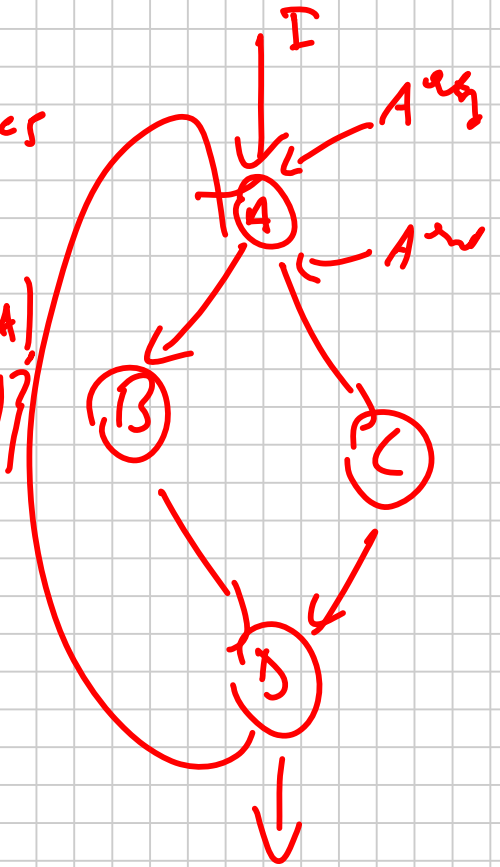
EQUATION SYSTEM FOR THE COLLECTION OF TRACE SEMANTICS

$$T(A^{exit}) = \{ t.c.(A, \ell).c' \mid \begin{array}{l} c' = \text{up}(c, A) \\ \ell = \text{hit}(c, A) \end{array} \}$$

$$T(A^{entry}) = T(D^{exit}) \cup T(I)$$

⋮

$$T(I) = \{ c \mid c \in C \}$$



$$\text{PERSISTENT}(T, \nu) = \forall \ell \in L. \forall t \in T(\ell). \forall i. (m_i, h_i) = (\nu, \text{min})$$

$$c_1(m_1, h_1) c_2(m_2, h_2) \dots \rightarrow \forall j. i. (m_j, h_j) \neq (\nu, \text{min})$$

SCOPES

```
while (...) {  
  [ for (...) {  
    if (. )  
    else  
  }  
}  
for ( . ) {  
  .  
  .  
  .  
}
```

ABSTRACTIONS FOR P(TRACE)

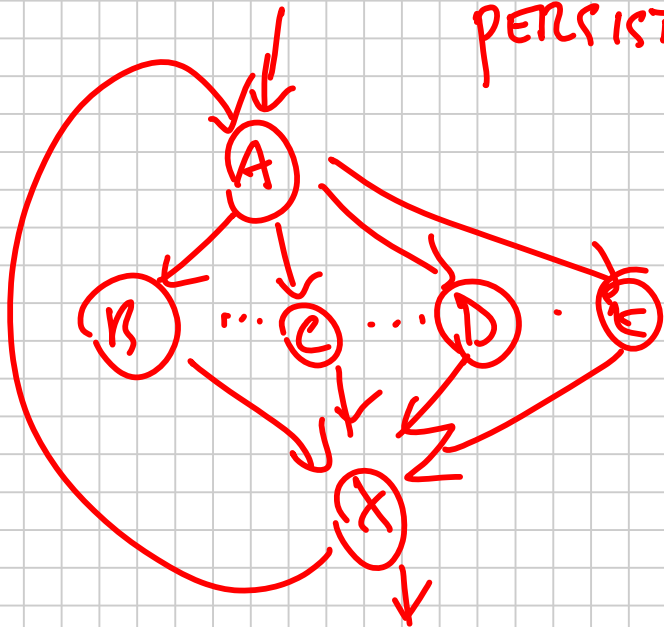
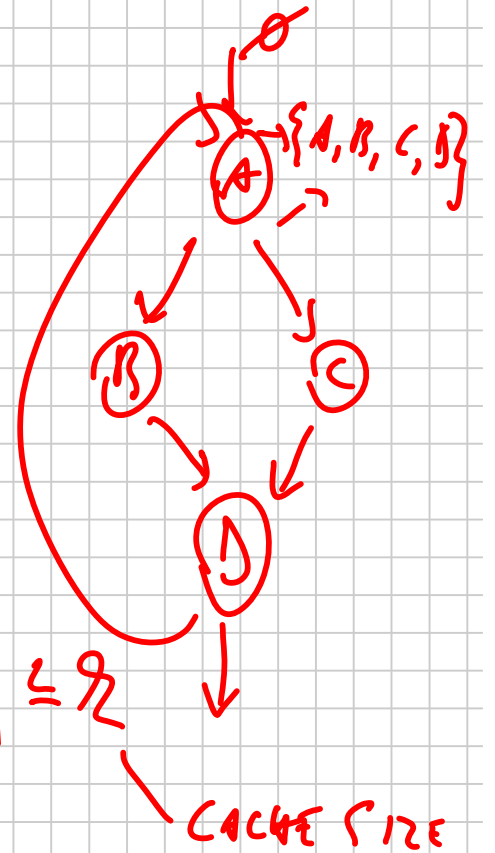
CONFLICT SET DOMAIN

$$CS = \mathcal{P}(M)$$

$$cs_1 \cup cs_2 := cs_1 \cup cs_2$$

$$\widehat{up}(cs, m) = cs \cup \{m\}$$

$$\widehat{PERSISTENT}(T, M) = \forall l \in L. |T(l)| \leq \mathcal{K}$$



\Rightarrow NOTHING IS DECLARED PERSISTENT

CONDITIONAL MUST ANALYSIS

$$CMUST = M \rightarrow \{ \top, \perp, \lambda, \omega \}$$

CONCRETIZATION?

$$\gamma(n, m) = \{ x \stackrel{c_m}{=} \mid \forall i. (m_i, h_i) = (n, ?) \rightarrow c_m(n) \leq m_i \}$$

MEMBER
TRACK

CONDITIONAL
UPPER BOUND
ON \mathbb{B}^k

$$\gamma(c_m) = \bigcap_{n \in M} \gamma(n, c_m(n))$$

JOIN?

$$c_{m_1} \sqcup c_{m_2} = \lambda n \in M : \max \{ c_{m_1}(n), c_{m_2}(n) \}$$

$$\text{INITIALLY : } c_{m_I} = \lambda n \in M : \top$$

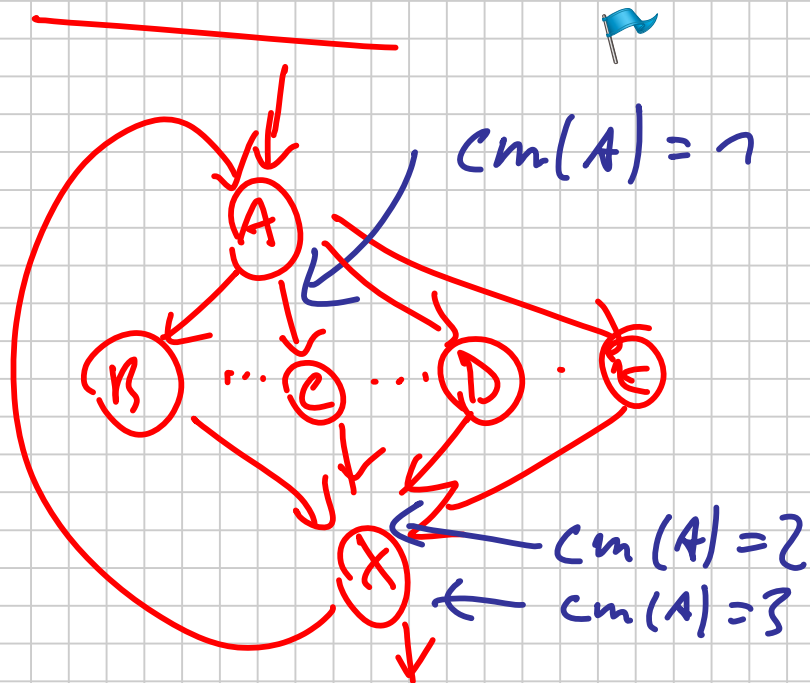
CONDITIONAL MUSEF ANALYSIS

UPDATE?

$$\widehat{up}(cm, v) = \lambda v' \in M.$$

$$\left\{ \begin{array}{l} \top : v' = v \\ \textcircled{cm(v') + 1} : \text{else if } cm(v') < h \\ \infty : \text{else} \end{array} \right.$$

EXAMPLE



$$\widehat{PERSISTENT}(T, v) = \forall l \in L. T(l)(v) \in l$$

BLOCK-WISE CONFLICT SET

$$BCS = M \rightarrow P(M)$$

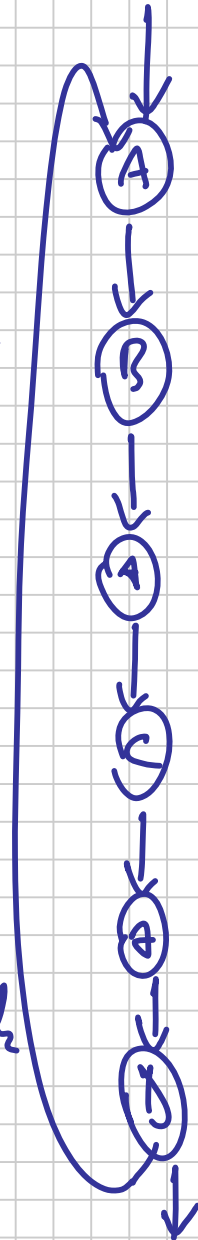
UPDATE ?

$$\hat{m}_i(nes, n) = \lambda n' \in M. \begin{cases} \emptyset : n' = n \\ nes(n') \cup \{n\} \end{cases}$$

JOIN ?

$$nes_1 \sqcup nes_2 = \lambda n' \in M. nes_1(n') \cup nes_2(n')$$

$$PERSISTENT(T, n) = \forall l \in L. |nes(n)| < l$$



CONFLICT SET DOMAIN
 $\{A, B, C, D\}$