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Exploring the Landscape of SSA-based Program Representations

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What is a Mathematician?

- A machine that converts coffee into theorems
- Beer-related variations exist as well
- More specifically
  - Defines something
  - Derives properties thereof via theorems and proofs
  - With luck, a useful application is found eventually
  - With a lot of luck, the application is found before said mathematician dies

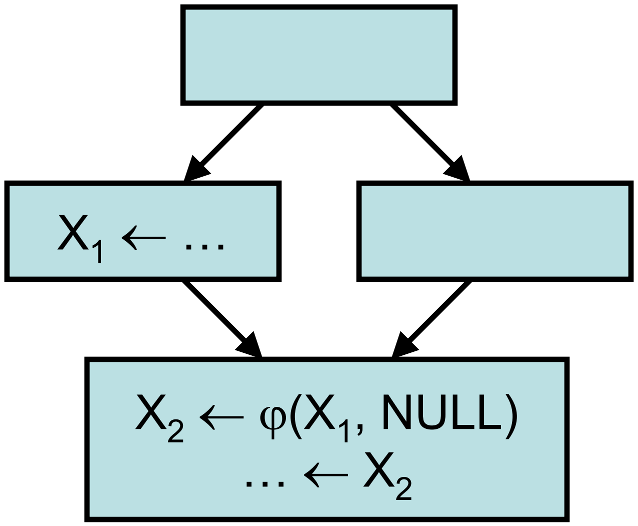
Disclaimers

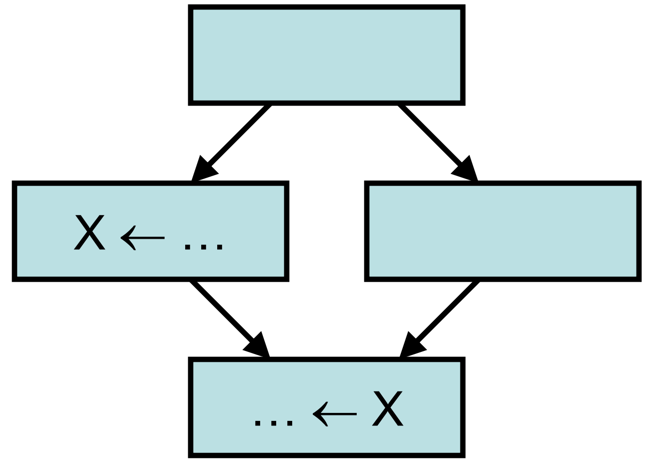
- Some of this talk has not been peer reviewed or published
- Conjectures abound

I Assume…

- You know what is SSA Form, and that you care
- This talk is not about SSA-based register allocation…
  - Except for the fact that, somehow, it is…
- “SSA Form” implies “Pruned SSA Form”
  - See above
  - DCE converts minimal or semi-pruned to pruned
- “SSA Form” implies Strict SSA
  - See the next slide…

Strict vs. Non-strict SSA Form





- Each definition dominates each use
- Arguably, we can eliminate this φ-function

- Fewer φ-functions
- Lose properties involving dominance, chordal interference graphs, etc.

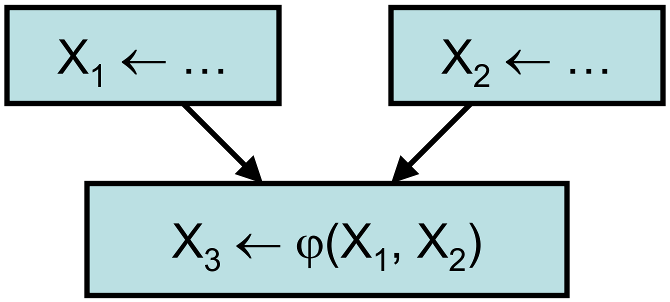
Key Points of SSA Form

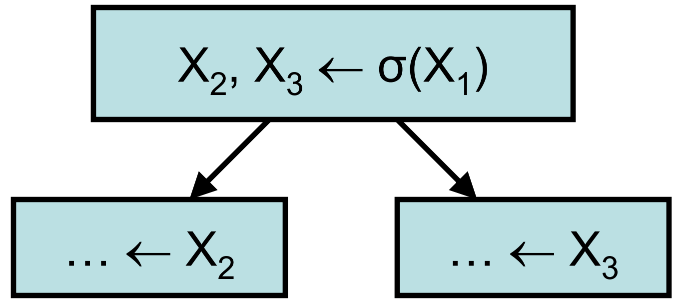
- Each definition dominates each use…
  - And each point where each variable is live
- Each variable live range is a subtree of the dominator tree
- A chordal graph is the intersection graph of a set of subtrees of a tree
  - $O(|V| + |E|)$-time algorithms for
    - coloring, clique, independent set, clique partition
- Should we do register allocation in SSA Form?

SSA Form is Plural

- You are probably familiar with φ-functions…
  - And Cytron et al.’s SSA construction method…
  - And maybe a few other equivalent construction methods too…
- φ-functions are just a way to split variable live ranges at convergence points in the control flow graph
  - With very specific parallel copy semantics
- Why stop there?

Other Ways to Split





Instr-1

$(Y_1, \ldots, Y_n) \leftarrow$ Parallel Copy$(X_1, \ldots, X_n)$

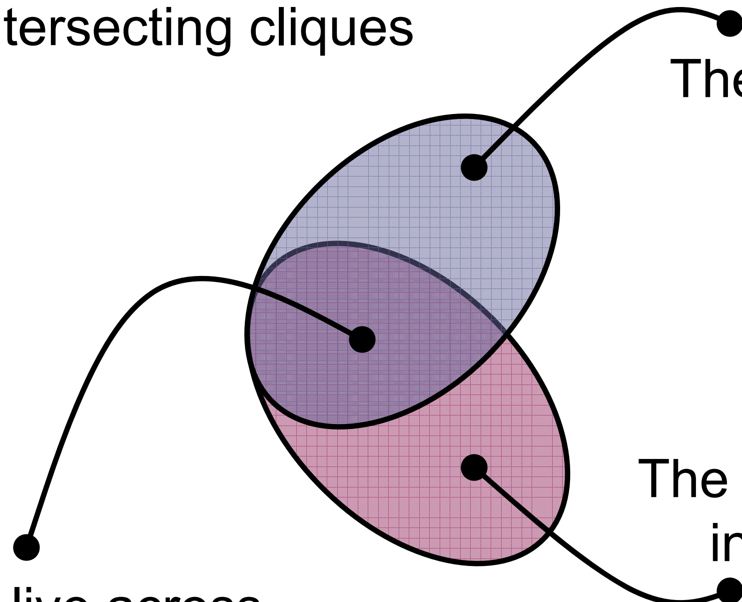
Instr-2

Elementary Form

- Split each variable at every place where it is live
  - φ-functions for all variables live at a merge point
  - σ-functions for all variables live at split points
  - Parallel copies for each variable live between two instructions
- Elementary graphs
  - Each connected component is the interference graph of one instruction
    - Technically, a clique substitution of P_3
  - A subclass of chordal graphs
    - Stronger theoretical properties than chordal graphs
  - Details will be provided in Jens Palsberg's talk on puzzle solving

The Interference Graph for One Instruction

- Two intersecting cliques



The variables whose lifetimes die at the instruction

The variables defined by the current instruction that become live

Variables live across the instruction

Conjecture Time…

- Let CHO be the class of chordal graphs
- Let ELEM be the class of elementary graphs
- Obviously, ELEM $\subset$ CHO

Problem 1

- Let X be a class of graphs such that
  - ELEM $\subseteq$ X $\subseteq$ CHO

- I want an SSA-based representation whose interference graph belongs to class X, because X has some favorable property

- **Answer**: Build Elementary Form
  - You get the property you want, and more…
  - If you care about the number of φ-functions, σ-functions, and parallel copies, reformulate the problem

Problem 2

- Let X, Y be classes of graphs such that
  - ELEM $\subseteq$ X $\subset$ Y $\subseteq$ CHO
- I want an SSA-based representation whose interference graph belongs to class Y, because Y has some favorable property
- I do not want to build an SSA-based representation whose interference graph belongs to class X, because doing so will introduce more φ-function, σ-functions, and parallel copies than I want to deal with
- **Conjecture(s)**:
  - An "efficient" algorithm exists to do this
  - The algorithm is sufficiently general for any pair of classes X and Y as defined above.

Problem 3

- Let X be a class of graphs such that
  - ELEM $\subseteq$ X $\subseteq$ CHO
- I want an SSA-based representation whose interference graph belongs to class X, because X has some favorable property
- I want to ensure this algorithm inserts the minimal number of φ-functions, σ-functions, and parallel copies
  - i.e., I won't settle for Elementary Form
- **Conjecture**:
  - An "efficient" algorithm exists to do this

More Rambling…

- Elementary graphs appear to be a hard lower bound
  - Given the instruction sets of today's processors
- No rule says that the upper bound in the preceding problem statements must be chordal graphs
  - Weakly chordal graphs
  - Perfect graphs
  - Any graph?
- Going beyond chordal graphs may require us to relax the notion of "efficient algorithm"
  - Last I heard, perfect graph recognition takes $O(|V|^9)$ time.

Limitations

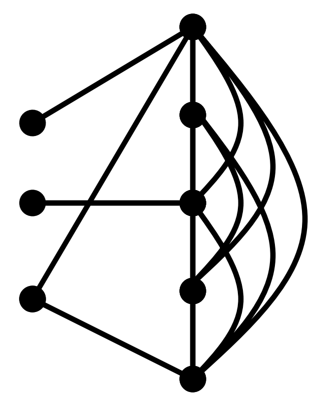
- There are some classes of graphs that cannot be characterized as the interference graph of a program in any realistic SSA-based representation that we know of.

- Example: Split graphs

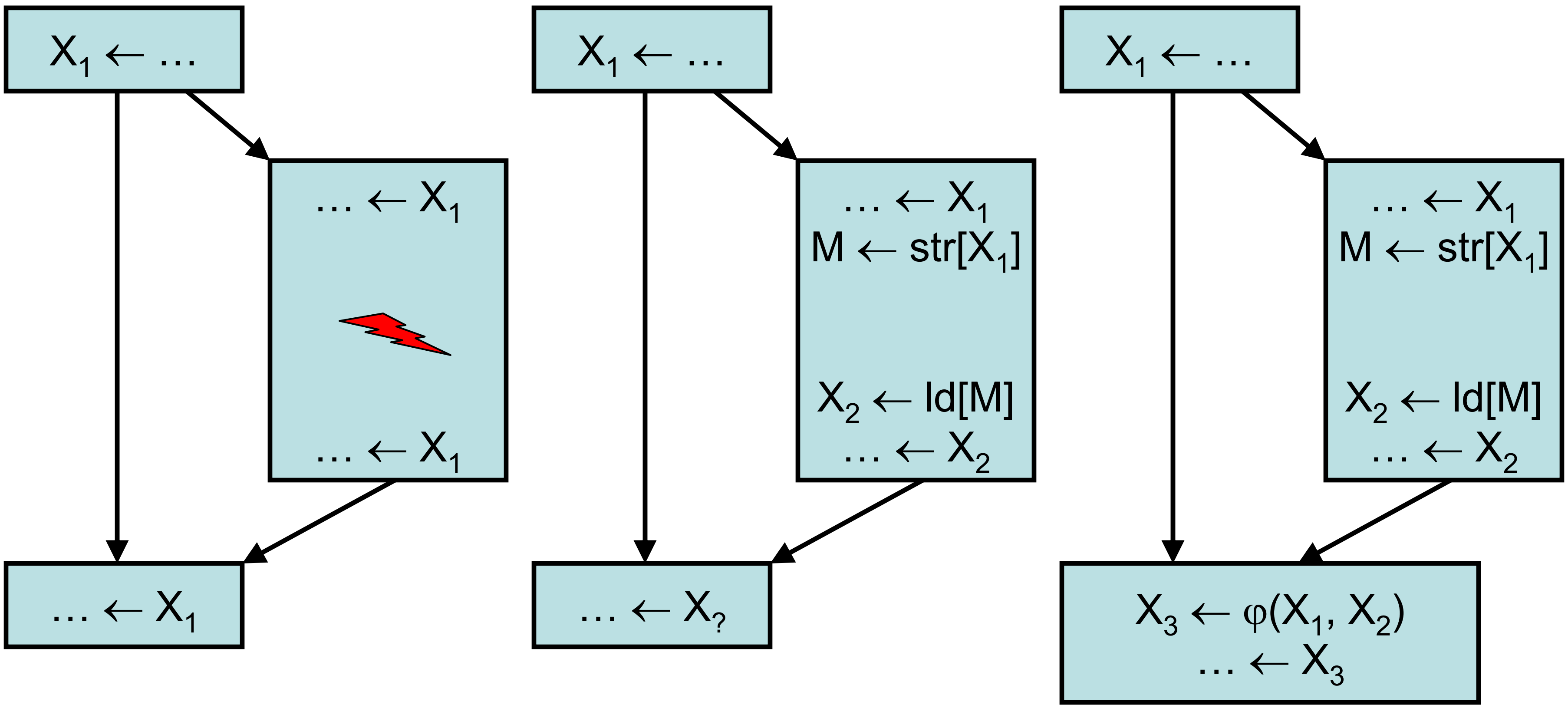
  – A chordal graph whose vertices can be partitioned into an independent set and a clique

  – Or, equivalently, the class of chordal graphs whose complements are chordal

  – Easy to find interference graphs for one instruction that are not Split graphs



Spilling Does Not Preserve the Cytron et al. SSA Form You Know



Key Points of SSA-based Spilling

- For simplicity, I ignore the finer details of spill code placement
- The issue of rebuilding SSA Form does not arrive under a spill-everywhere model
  - So, assume we don’t spill everywhere
  - Good idea, as this reduces the amount of spill code
- Every use of a variable in SSA Form is the placeholder for a potential new definition, after spilling
  - The load placed before the use is the new definition

Cytron et al.'s SSA Construction Algorithm

- D_v – the set of basic blocks containing definitions of v
- IDF(…) – the iterated dominance frontier of a set of basic blocks
- Place φ-functions for v at the entry of every basic block in $IDF(D_v)$
  - Yields minimal form
  - Filtering yields semi-pruned form
    - See [Briggs et al., SPE 1998]
  - Dead code elimination converts to pruned form
    - Folklore, but easy to prove

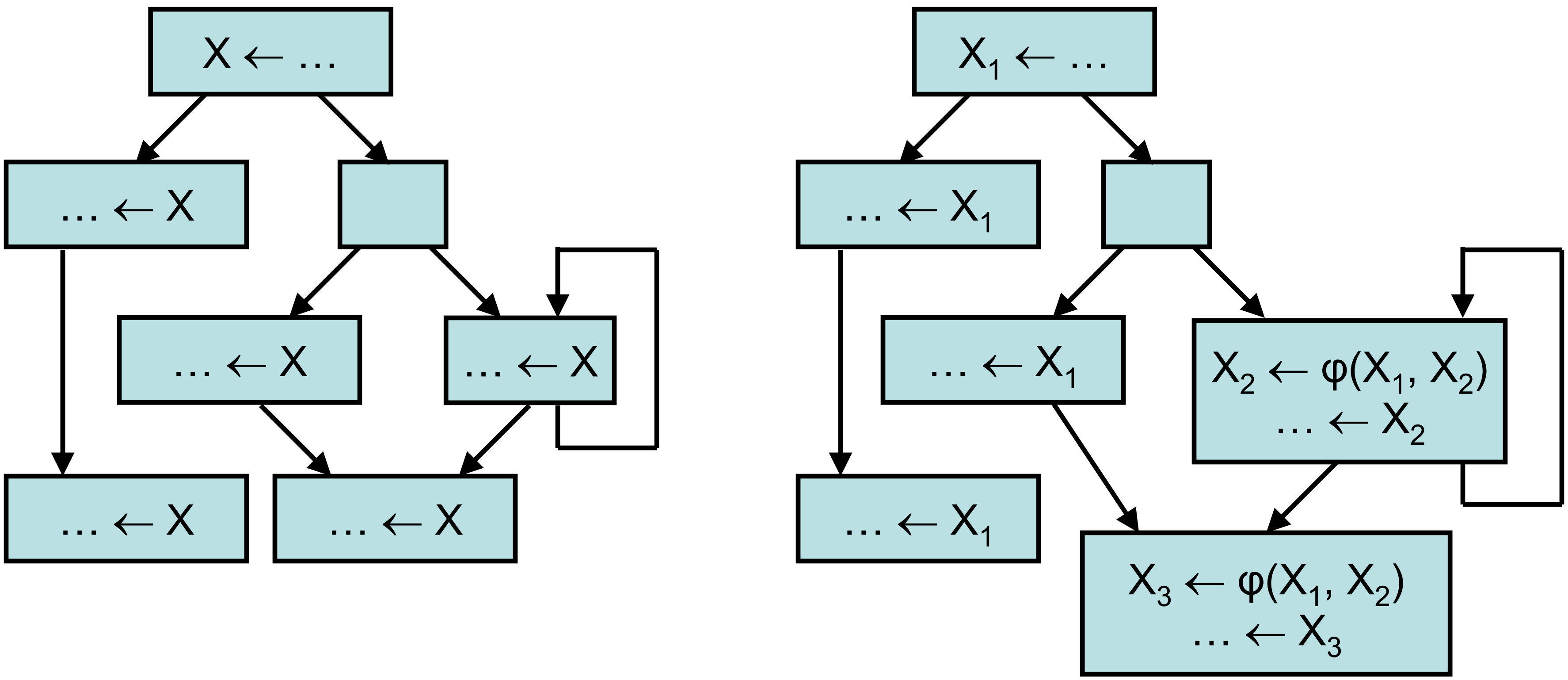
SSRO: Yet-another SSA Variant (Acronym to be Explained Later)

- D_v – the set of basic block containing definitions of v
- DU_v – the set of basic blocks containing <u>definitions or uses</u> of v

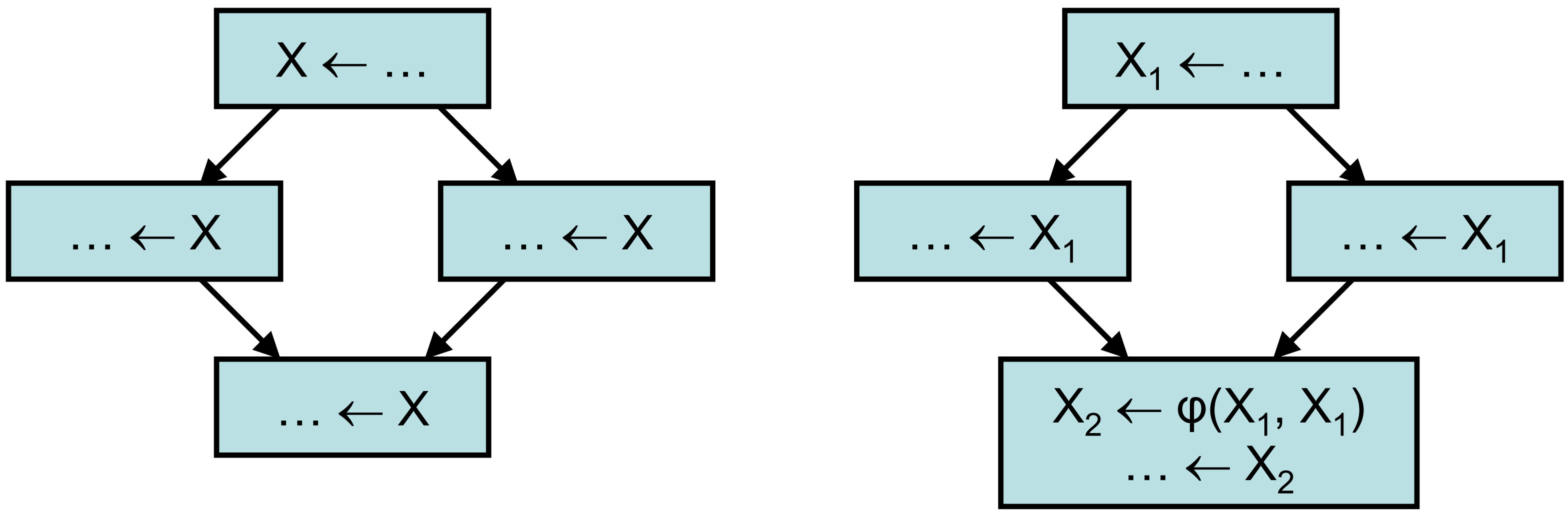
- IDF(…) – the iterated dominance frontier of a set of basic blocks

- Place φ-functions for v at the entry of every basic block in $IDF(DU_v)$
  - In contrast, $IDF(D_v)$ for SSA Form
  - Minimal, semi-pruned, pruned variants exist

SSA on the Left / SSRO on the Right



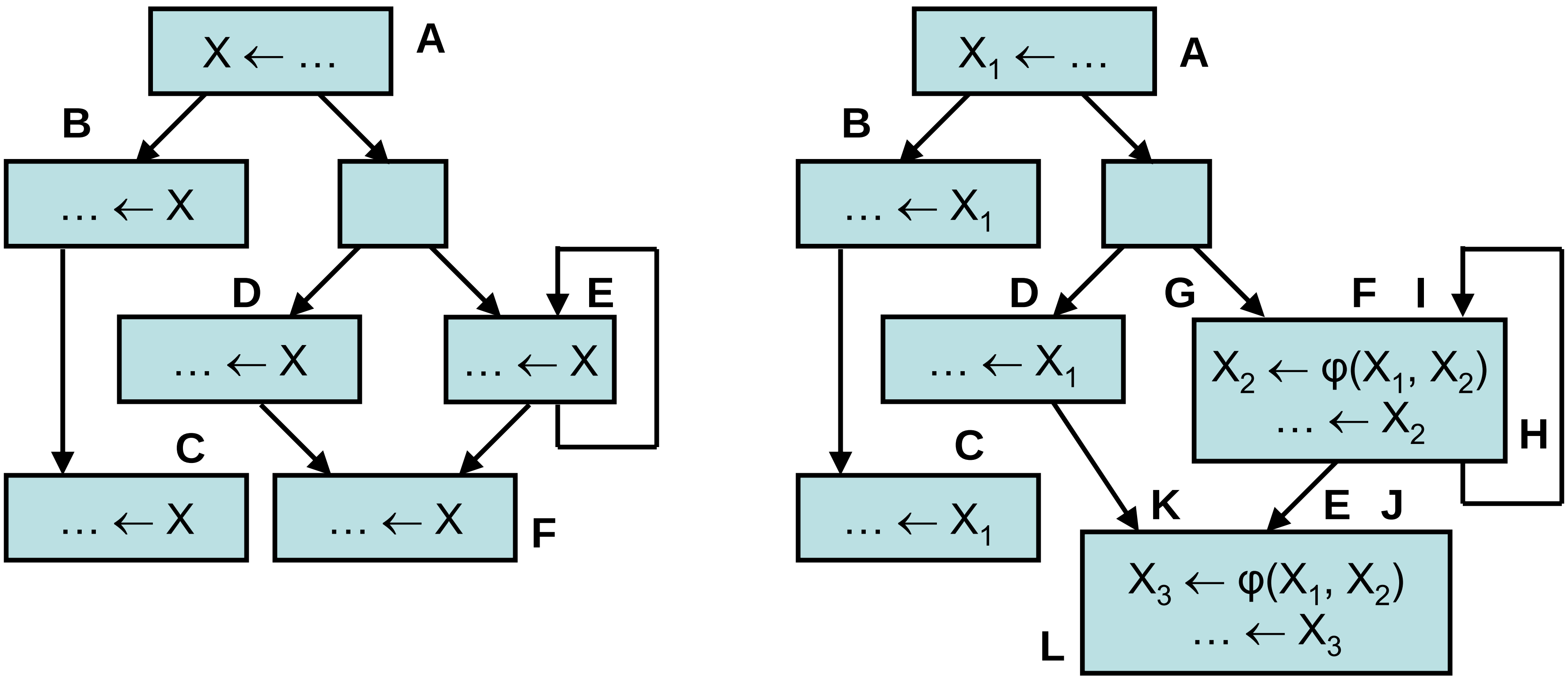
SSA on the Left / SSRO on the Right



Definitions

- **Occurrence** – a definition or use of a variable
- An occurrence O_1 of variable v **reaches** a second occurrence O_2 of v if there is a path in the CFG from O_1 to O_2 that does not pass through any other occurrence of v.
- $ReachOcc_v[O_i]$ – the set of reaching occurrences of v that reach O_i

Reaching Definitions



$ReachOcc_x[B] = \{A\}$

$ReachOcc_x[C] = \{B\}$

$ReachOcc_x[D] = \{A\}$

$ReachOcc_x[E] = \{A, E\}$

$ReachOcc_x[F] = \{D, E\}$

$ReachOcc_{x1}[B] = \{A\}$

$ReachOcc_{x1}[C] = \{B\}$

$ReachOcc_{x1}[D] = \{A\}$

$ReachOcc_{x1}[E] = \{D\}$

$ReachOcc_{x1}[F] = \{A\}$

$ReachOcc_{x2}[H] = \{G\}$

$ReachOcc_{x2}[I] = \{H\}$

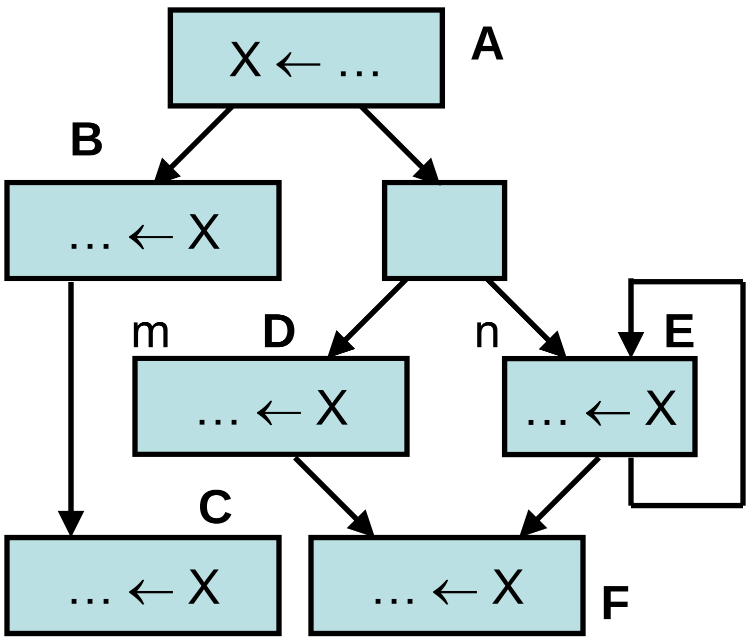
$ReachOcc_{x2}[J] = \{H\}$

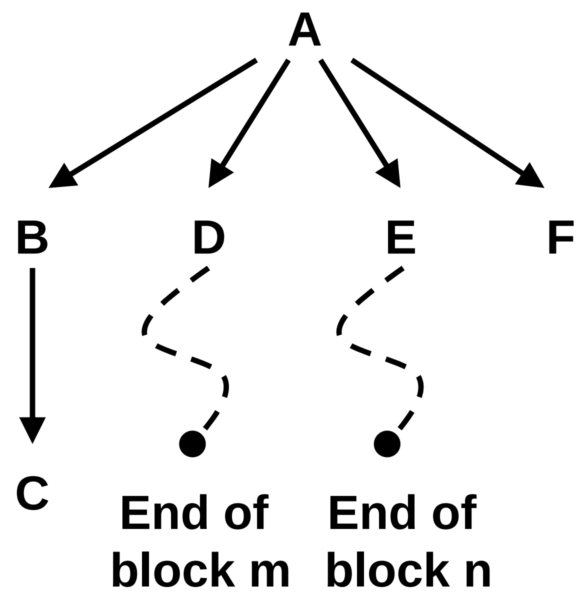
$ReachOcc_{x3}[L] = \{K\}$

The Def-Use Tree

- In SSA Form, the definition of each variable dominates all of its uses
- Organize definitions and uses as a tree
  - idom O_i – immediate dominating occurrence of use O_i
  - i.e., the parent of O_i in the DU-tree

Leaves and Death Points





$ReachOcc_x[B] = \{A\}$

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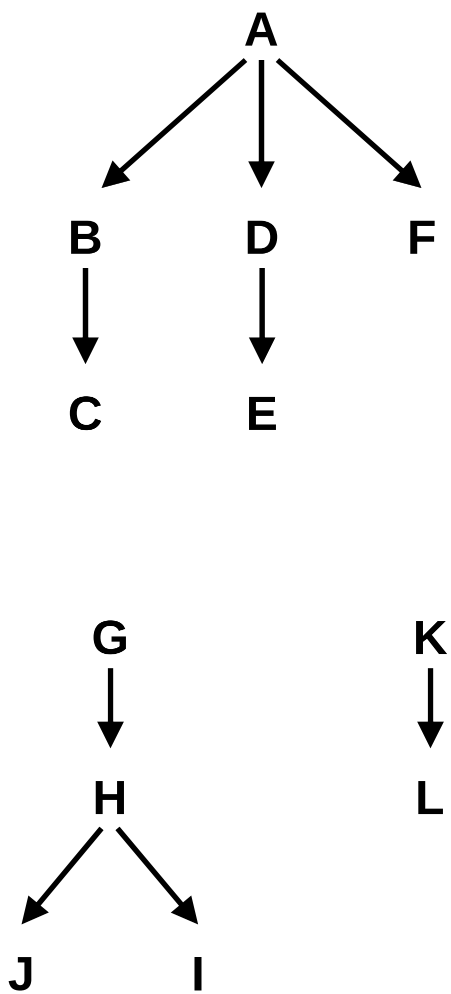
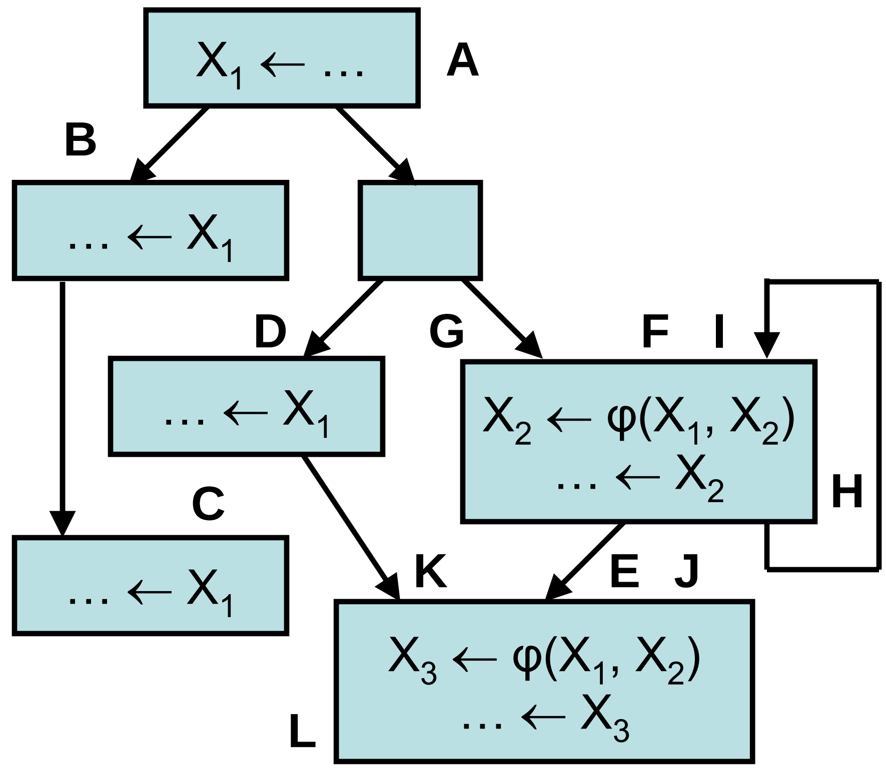
Leaves and Death Points





$\text{ReachOcc}_{x1}[B] = \{A\}$

$\text{ReachOcc}_{x1}[C] = \{B\}$

$\text{ReachOcc}_{x1}[D] = \{A\}$

$\text{ReachOcc}_{x1}[E] = \{D\}$

$\text{ReachOcc}_{x1}[F] = \{A\}$

$\text{ReachOcc}_{x2}[H] = \{G\}$

$\text{ReachOcc}_{x2}[I] = \{H\}$

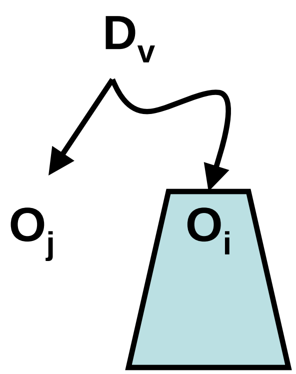
$\text{ReachOcc}_{x2}[J] = \{H\}$

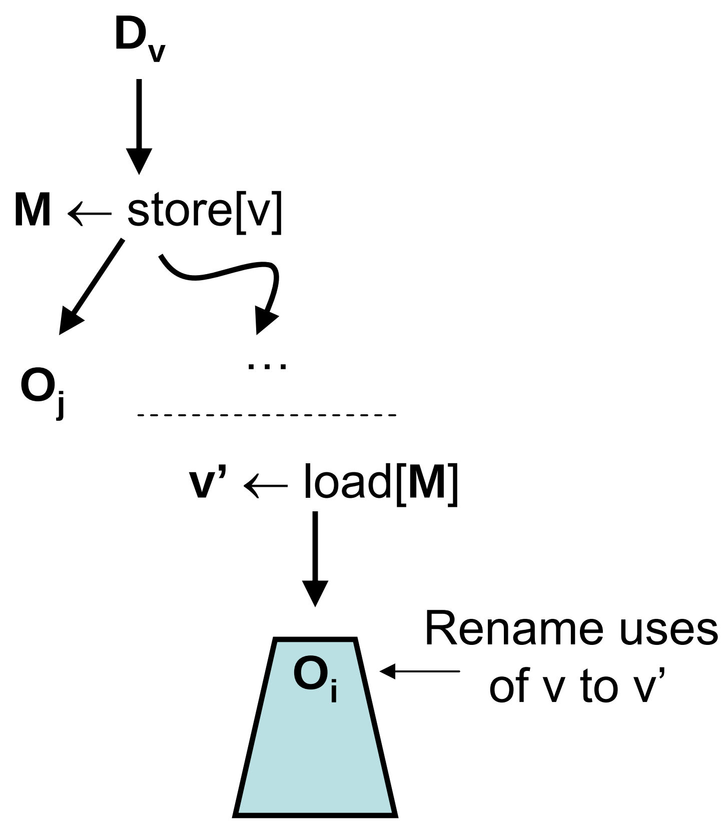
$\text{ReachOcc}_{x3}[L] = \{K\}$

The Static Single Reaching Occurrence (SSRO) Property

- **Theorem**: In SSRO Form, the set of reaching occurrences for each use is a singleton
  - Specifically, $ReachOcc_x[O_i] = \{idom\ O_i\}$
- **Theorem**: In SSRO Form, the death point of each variable corresponds to a leaf in the def-use tree
  - If not, there is a path from O_i to itself, so $|ReachOcc_x[O_i]| > 1$
  - Contradicts the theorem above

Spilling Under SSRO Form





Spilling Under SSRO Form

- **Theorem**: There is no need to insert any additional φ-functions if spilling is applied under SSRO Form.
  - For each use of v, $\text{ReachOcc}_v[O_i]$ = {idom O_i}
  - Any path from occurrence O_j to use O_i must pass through idom O_i
- Practical Issues
  - Simplifies process of SSA-based register allocation
  - Additional φ-functions suggest…
    - Live range splitting on a finer granularity than SSA Form
    - Probably better for spilling, but worse for coalescing

Summary: Key Properties of SSRO Form

- The set of reaching occurrences of each use is a singleton
- Each death point of a variable corresponds to a use
  - Organize the definition and uses of each variable into a tree
  - Each death point is a leaf, and vice-versa
- No additional φ-functions must be inserted after spilling
  - i.e., a procedure in SSRO Form remains in SSRO Form after spilling
- Like SSA Form, the interference graph is chordal
  - i.e., given a chordal graph, I can construct an SSRO Form procedure whose interference graph is the same as the given graph.

Going Interprocedural

- It is possible to build a whole program representation such that the interprocedural interference graph is chordal
  - Only works if I can resolve all function pointers in advance
  - Paper published at ICCAD 2007
- Extensions are necessary to extend the result to Elementary Form
  - I have worked them out in my head
  - Call it a conjecture for now

Recursive Calls

- How to handle variables live across calls in a recursive chain?
  - Pushed onto stack
  - Cannot use registers
- Call graph becomes a DAG
  - Strongly connected components – $O(|V| + |E|)$
  - Collapse each SCC into a single node

Local and Global Interference

- Local Interferences
  - Variables in the same procedure
  - Overlapping lifetimes
- Global Interferences
  - Variables live across procedure calls
  - Interferences are transitive

X ← ...

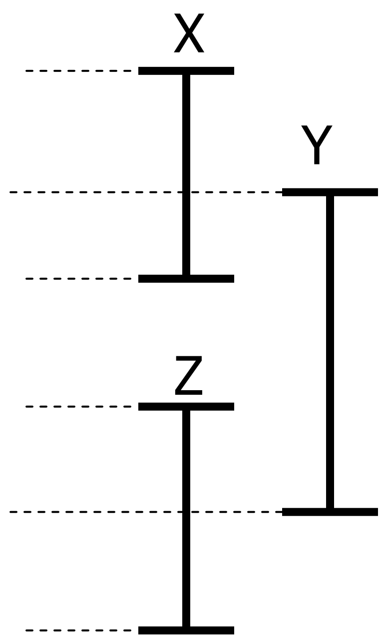
Y ← ...

... ← X

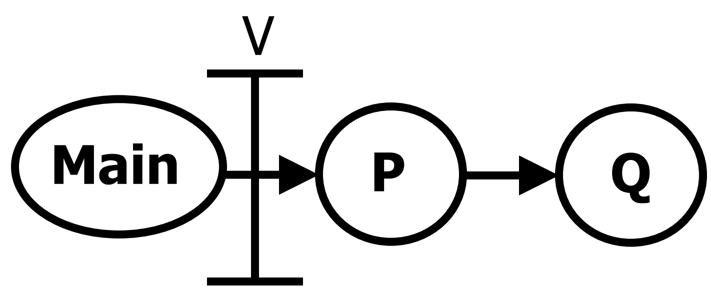
Z ← ...

... ← Y

... ← Z



Main:	P:
V ← ...	...
Call P	Call Q
... ← V	...



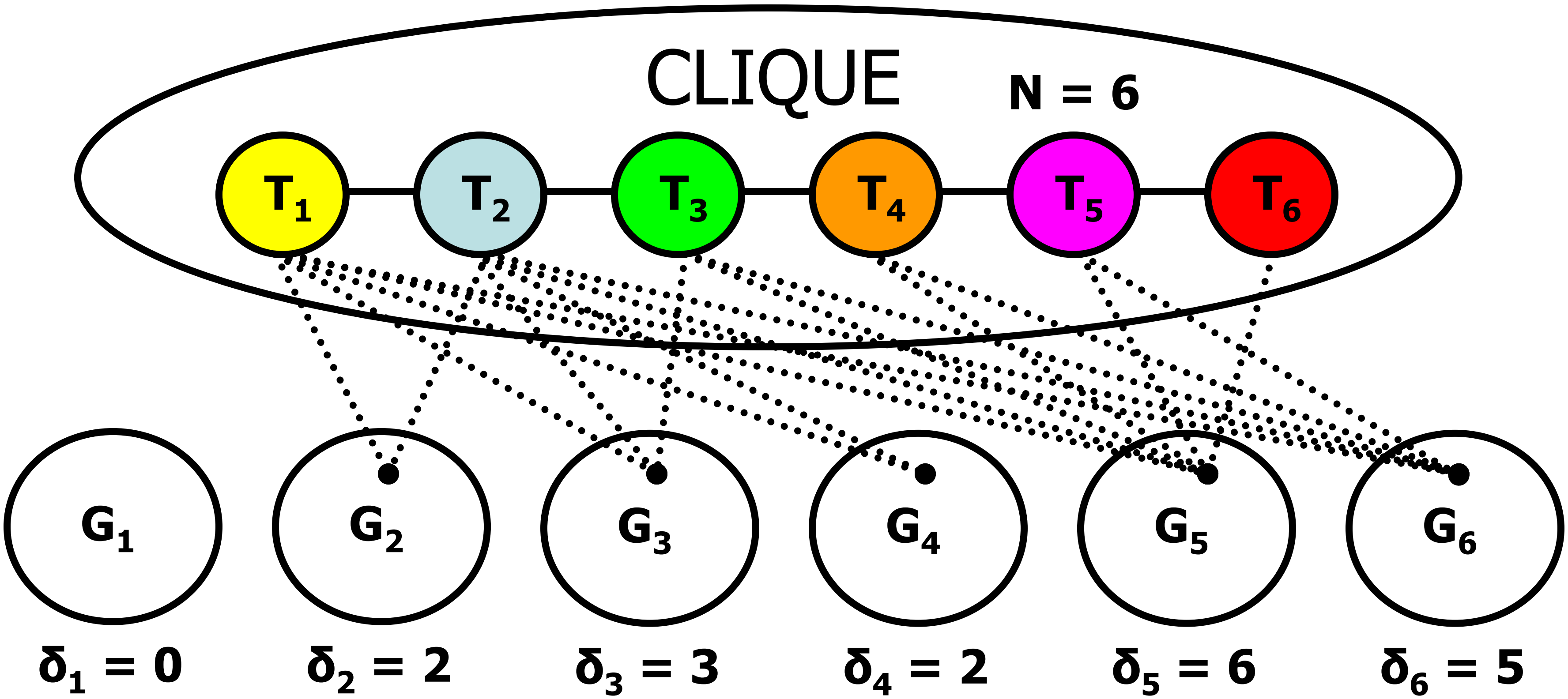
Launch and Landing Pads

- When P_i is called
  - The maximum stack size is $m = \delta_i$
    - Taken among all paths in the call graph leading to P_i
  - Global registers $T_1 \ldots T_m$ store variables live across calls in the chain

- P_i calls P_j at call point c_k
  - $L(c_k)$ – set of variables live across the call
  - Let $n = |L(c_k)|$ be the number of variables

- Launch and Landing Pads
  - Parallel copy $(T_{m+1} \ldots T_{m+n}) \leftarrow \psi(L(c_k))$ inserted before the call
  - Parallel copy $L(c_k) \leftarrow \psi^{-1}(T_{m+1} \ldots T_{m+n})$ inserted after the call

The Interprocedural Interference Graph is Chordal



T_j interferes with each local variable in G_i

G_i is chordal

Conclusions

- If you think in terms of classes of interference graphs, there are a wide variety of SSA-based representations that have yet to be explored
  - Not clear if they are useful for register allocation
  - Not clear if they provide superior facilities for dataflow analysis
- SSRO Form is somehow orthogonal to the above
  - I invented it when thinking about spilling under SSA
  - Eliminates the need to insert additional φ-functions after spilling
- Interprocedural extensions
  - Only if we can resolve function pointers