Lexical Analysis

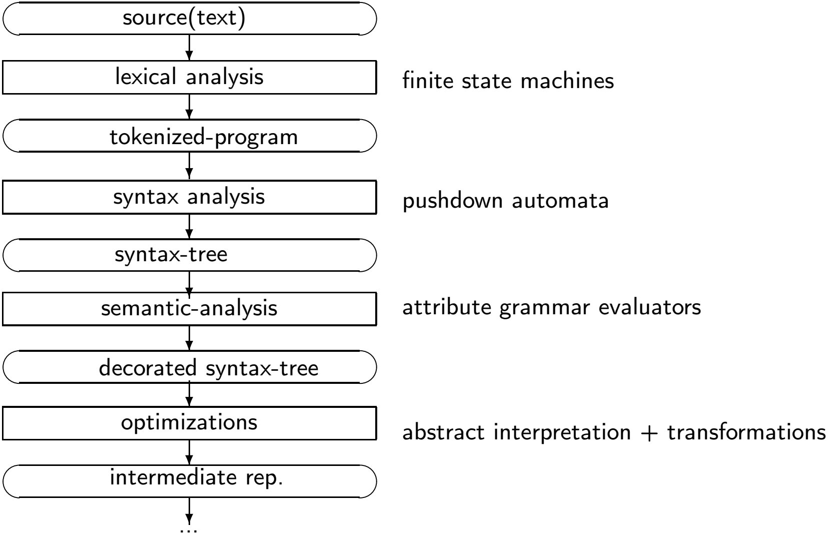
Reinhard Wilhelm
Universität des Saarlandes
wilhelm@cs.uni-saarland.de
and
Mooly Sagiv
Tel Aviv University
sagiv@math.tau.ac.il

23. Oktober 2013

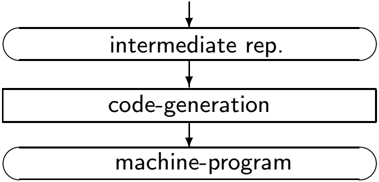
Subjects

- Role of lexical analysis
- Regular languages, regular expressions
- Finite state machines
- From regular expressions to finite state machines
- A language for specifying lexical analysis
- The generation of a scanner
- Flex

"Standard" Structure



"Standard" Structure cont'd



tree automata + dynamic programming + $\cdots$

Lexical Analysis (Scanning)

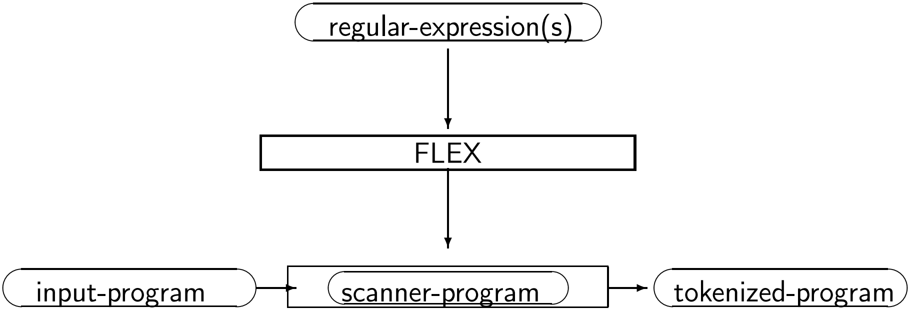
- Functionality

  Input: program as sequence of characters
  Output: program as sequence of symbols (tokens)
- Produce listing
- Report errors, symbols illegal in the programming language
- Screening subtask:
  - Identify language keywords and standard identifiers
  - Eliminate "white-space", e.g., consecutive blanks and newlines
  - Count line numbers
  - Construct table of all symbols occurring

Automatic Generation of Lexical Analyzers

- The symbols of programming languages can be specified by regular expressions.
- Examples:
  - `program` as a sequence of characters.
  - `((alpha | {_} ) (alpha | digit | {_} )*)` for C identifiers
  - `((‘‘/ *‘‘ until ‘‘* /‘‘) | ( // until NL ))` for C comments
- The recognition of input strings can be performed by a finite state machine.
- A table representation or a program for the automaton is automatically generated from a regular expression.

Automatic Generation of Lexical Analyzers cont'd



Notations

A language, L, is a set of words, x, over an alphabet, Σ.

$a_1 a_2 \ldots a_n$,	a word over Σ, $a_i \in \Sigma$
ε	The empty word
Σ^n	The words of length n over Σ
Σ^*	The set of finite words over Σ
Σ^+	The set of non-empty finite words over Σ
$x.y$	The concatenation of x and y

Language Operations

$L_1 \cup L_2$		Union
$L_1 L_2$	$= \{x.y \mid x \in L_1, y \in L_2\}$	Concatenation
$\overline{L}$	$= \Sigma^* - L$	Complement
L^n	$= \{x_1 \ldots x_n \mid x_i \in L, 1 \leq i \leq n\}$	
L^*	$= \bigcup_{n \geq 0} L^n$	Closure
L^+	$= \bigcup_{n \geq 1} L^n$	

Regular Languages

Defined inductively

- $\emptyset$ is a regular language over Σ
- $\{\varepsilon\}$ is a regular language over Σ
- For all $a \in \Sigma$, $\{a\}$ is a regular language over Σ
- If R_1 and R_2 are regular languages over Σ, then so are:
  - $R_1 \cup R_2$,
  - $R_1 R_2$, and
  - R_1^*

Regular Expressions and the Denoted Regular Languages

Defined inductively

- $\underline{\emptyset}$ is a regular expression over Σ denoting $\emptyset$,
- $\underline{\varepsilon}$ is a regular expression over Σ denoting $\{\varepsilon\}$,
- For all $a \in \Sigma$, a is a regular expression over Σ denoting $\{a\}$,
- If r_1 and r_2 are regular expressions over Σ denoting R_1 and R_2, resp., then so are:
  - $\underline{(}r_1\underline{|}r_2\underline{)}$, which denotes $R_1 \cup R_2$,
  - $\underline{(}r_1 r_2\underline{)}$, which denotes $R_1 R_2$, and
  - $\underline{(}r_1\underline{)}\underline{^*}$, which denotes R_1^*.
- Metacharacters, $\underline{\emptyset}, \underline{\varepsilon}, \underline{(}, \underline{)}, \underline{|}, \underline{*}$ don't really exist, are replaced by their non-underlined versions. Attention: Clash between characters in Σ and metacharacters $\{\underline{(}, \underline{)}, \underline{|}, \underline{^*}\}$

Example

Expression	Language	Example Words
$a \mid b$		
ab^*a		
$(ab)^*$		
$abba$		

Example

Expression	Language	Example Words
$a \vert b$	$\{a, b\}$	a, b
ab^*a	$\{a\}\{b\}^*\{a\}$	$aa, aba, abba, abbba, \ldots$
$(ab)^*$	$\{ab\}^*$	$\varepsilon, ab, abab, \ldots$
$abba$	$\{abba\}$	$abba$

Regular Expressions for (Sets of) Symbols (Tokens)

integer constants

float constants

C identifiers

strings

comments

matching-parentheses

Automata

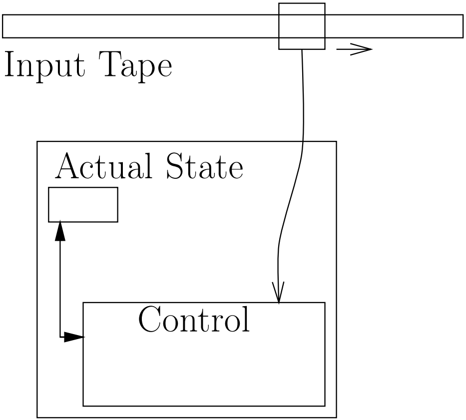
In the following, we will meet different types of automata.

Automata

- process some input, e.g. strings or trees,
- make transitions from configurations to configurations;
- configurations consist of (the rest of) the input and some contents of some memory;
- the memory may be small, just one variable with finitely many values,
- but the memory may also be able to grow without bound, adding and removing values at one of its ends;
- the type of memory an automaton has determines its ability to recognize a class of languages,
- in fact, the more powerful an automaton type is, the better it is in rejecting input.

Finite State Machine

The simplest type of automaton, its memory consists of only one variable, which can store one out of finitely many values, its states,



A Non-Deterministic Finite State Machine (NFSM)

$M = \langle \Sigma, Q, \Delta, q_0, F \rangle$ where:

- Σ — finite alphabet
- Q — finite set of states
- $q_0 \in Q$ — initial state
- $F \subseteq Q$ — final states
- $\Delta \subseteq Q \times (\Sigma \cup \{\varepsilon\}) \times Q$ — transition relation

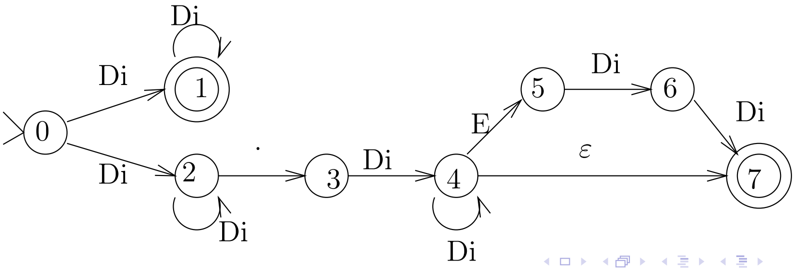
May be represented as a transition diagram

- Nodes — States
- q_0 has a special "entry" mark
- final states doubly encircled
- An edge from p into q labeled by a if $(p, a, q) \in \Delta$

Example: Integer and (simplified) Float Constants

	Di $\in \{0, 1, \ldots, 9\}$	.	E	ε
0	{1,2}	$\emptyset$	$\emptyset$	$\emptyset$
1	{1}	$\emptyset$	$\emptyset$	$\emptyset$
2	{2}	{3}	$\emptyset$	$\emptyset$
3	{4}	$\emptyset$	$\emptyset$	$\emptyset$
4	{4}	$\emptyset$	{5}	{7}
5	{6}	$\emptyset$	$\emptyset$	$\emptyset$
6	{7}	$\emptyset$	$\emptyset$	$\emptyset$
7	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

$$q_0 = 0$$
$$F = \{1, 7\}$$



Finte State Machines — Scanners

Finite state machines

- get an input word,
- start in their initial state,
- make a series of transitions under the characters constituting the input word,
- accept (or reject).

Scanners

- get an input string (a sequence of words),
- start in their initial state,
- attempt to find the end of the next word,
- when found, restart in their initial state with the rest of the input,
- terminate when the end of the input is reached or an error is encountered.

Maximal Munch strategy

Find longest prefix of remaining input that is a legal symbol.

- first input character of the scanner — first “non-consumed” character,
- in final state, and exists transition under the next character: make transition and remember position,
- in final state, if there exists no transition under the next character: Symbol found,
- if actual state not final and there exists no transition under the next character: backtrack to last passed final state
  - There is none: Illegal string
  - Otherwise: Actual symbol ended there.

Warning: Certain overlapping symbol definitions will result in quadratic runtime: Example: $(a|a^*;)$

Other Example Automata

- integer constants
- float constants
- C identifiers
- strings
- comments

The Language Accepted by a Finite-State Machine

- $M = \langle \Sigma, Q, \Delta, q_0, F \rangle$
- For $q \in Q$, $w \in \Sigma^*$, (q, w) is a configuration
- The binary relation step on configurations is defined by:

$$(q, aw) \vdash_M (p, w)$$

  if $(q, a, p) \in \Delta$
- The reflexive transitive closure of $\vdash_M$ is denoted by $\vdash_M^*$
- The language accepted by M

$$L(M) = \{w \mid w \in \Sigma^* \mid \exists q_f \in F : (q_0, w) \vdash_M^* (q_f, \varepsilon)\}$$

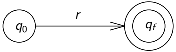
From Regular Expressions to Finite State Machines

Theorem

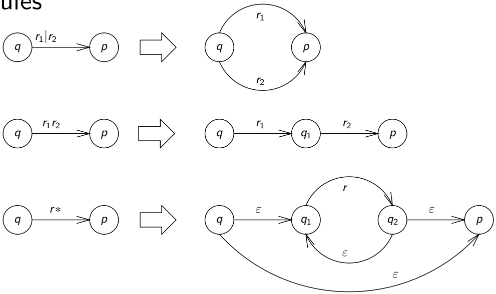
(i) For every regular language R, there exists an NFSM M, such that $L(M) = R$.
(ii) For every regular expression r, there exists an NFSM that accepts the regular language defined by r.

A Constructive Proof for (ii) (Algorithm)

- A regular language is defined by a regular expression r
- Construct an "NFSM" with one final state, q_f, and the transition



- Decompose r and develop the NFSM according to the following rules



until only transitions under single characters and ε remain.

Examples

- $a(a|0)^*$ over $\Sigma = \{a, 0\}$
- C Identifiers
- Strings

Nondeterminism

- Several transitions may be possible under the same character in a given state
- ε-moves (next character is not read) may "compete" with non-ε-moves.
- Deterministic simulation requires "backtracking"

Deterministic Finite Automaton (DFSM)

- No ε-transitions
- At most one transition from every state under a given character, i.e. for every $q \in Q$, $a \in \Sigma$,

$$|\{q' \mid (q, a, q') \in \Delta\}| \leq 1$$

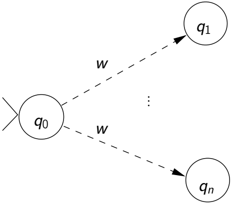
From Non-Deterministic to Deterministic State Machines

Theorem

For every NFSM, $M = \langle \Sigma, Q, \Delta, q_0, F \rangle$ there exists a DFSM, $M' = \langle \Sigma, Q', \delta, q'_0, F' \rangle$ such that $L(M) = L(M')$.

A Scheme of a Constructive Proof (Powerset Construction)

Construct a DFSM whose states are sets of states of the NFSM.
The DFSM simulates all possible transition paths under an input word in parallel.
Set of new states
$\{\{q_1, \ldots, q_n\} \mid n \geq 1 \wedge \exists w \in \Sigma^* : (q_0, w) \vdash_M^* (q_i, \varepsilon)\}$



The Construction Algorithm

Used in the construction: the set of ε-Successors,
$\varepsilon\text{-}SS(q) = \{p \mid (q, \varepsilon) \vdash_M^* (p, \varepsilon)\}$

- Starts with $q_0' = \varepsilon\text{-}SS(q_0)$ as the initial DFSM state.
- Iteratively creates more states and more transitions.
- For each DFSM state $S \subseteq Q$ already constructed and character $a \in \Sigma$, construct the a-successor of S

$$\delta(S, a) = \bigcup_{q \in S} \bigcup_{(q,a,p) \in \Delta} \varepsilon\text{-}SS(p)$$

  if non-empty
    add new state $\delta(S, a)$ if not previously constructed;
    add transition from S to $\delta(S, a)$.

- A DFSM state S is accepting (in F') if there exists $q \in S$ such that $q \in F$

Closure program

$$
\begin{array}{l}
\textbf{set}\langle \mathit{state}\rangle\ \mathrm{closure}(\textbf{set}\langle \mathit{state}\rangle\ S)\ \ \{ \\
\quad \textbf{set}\langle \mathit{state}\rangle\ \mathit{result} \leftarrow \emptyset; \\
\quad \textbf{list}\langle \mathit{state}\rangle\ \ W \leftarrow \mathrm{list\_of}(S); \\
\quad \mathit{state}\ \ q, q'; \\
\quad \textbf{while}\ \ (W \neq [])\ \ \{ \\
\quad\quad q \leftarrow \mathrm{hd}(W);\ \ W \leftarrow \mathrm{tl}(W); \\
\quad\quad \textbf{if}\ (q \notin \mathit{result})\ \ \{ \\
\quad\quad\quad \mathit{result} \leftarrow \mathit{result} \cup \{q\}; \\
\quad\quad\quad \textbf{forall}\ \ (q' : (q, \varepsilon, q') \in \Delta) \\
\quad\quad\quad\quad W \leftarrow q' :: W; \\
\quad\quad \} \\
\quad \} \\
\quad \textbf{return}\ \ \mathit{result}; \\
\}
\end{array}
$$

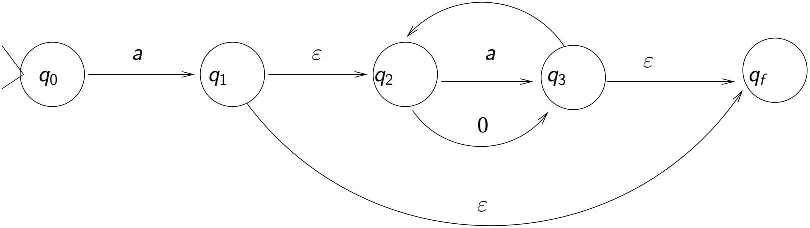
Function succState()

$$
\begin{array}{l}
\textbf{set}\langle \mathit{state}\rangle\ \text{succState}(\textbf{set}\langle \mathit{state}\rangle\ S, \mathit{symbol}\ \ x)\ \ \{ \\
\qquad \textbf{set}\langle \mathit{state}\rangle\ S' \leftarrow \emptyset; \\
\qquad \mathit{state}\ \ q, q'; \\
\qquad \textbf{forall}\ (q' : q \in S, (q, x, q') \in \Delta)\ \ S' \leftarrow S' \cup \{q'\}; \\
\qquad \textbf{return}\ \ \text{closure}(S'); \\
\qquad \}
\end{array}
$$

Powerset program

$$
\begin{array}{l}
\textbf{list}\langle\textbf{set}\rangle \mathit{state} \quad W;\\
\textbf{set}\langle \mathit{state}\rangle \quad S_0 \leftarrow \mathrm{closure}(\{q_0\});\\
\mathit{states} \leftarrow \{S_0\}; \quad W \leftarrow [S_0]; \mathit{trans} \leftarrow \emptyset;\\
\textbf{set}\langle \mathit{state}\rangle \quad S, S';\\
\textbf{while} \ \ (W \neq []) \ \ \{\\
\qquad S \leftarrow \mathrm{hd}(W); \quad W \leftarrow \mathrm{tl}(W);\\
\qquad \textbf{forall} \ (x \in \Sigma) \ \ \{\\
\qquad\qquad S' \leftarrow \mathrm{succState}(S, x);\\
\qquad\qquad \mathit{trans} \leftarrow \mathit{trans} \cup \{(S, x, S')\};\\
\qquad\qquad \textbf{if} \ \ (S' \notin \mathit{states}) \ \ \{\\
\qquad\qquad\qquad \mathit{states} \leftarrow \mathit{states} \cup \{S'\};\\
\qquad\qquad\qquad W \leftarrow W \cup \{S'\};\\
\qquad\qquad \}\\
\qquad \}\\
\}
\end{array}
$$

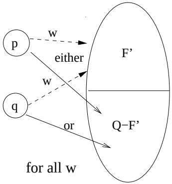
Example: $a(a|0)^*$



DFSM minimization

DFSM need not have minimal size, i.e. minimal number of states and transitions.
q and p are undistinguishable iff

for all words w $(q, w) \vdash_M^*$ and $(p, w) \vdash_M^*$ lead into either F' or $Q' - F'$.

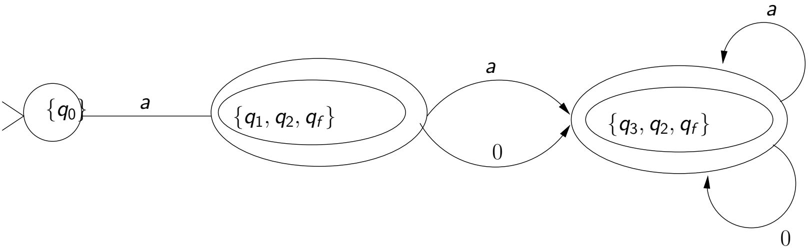


After termination merge undistinguishable states.

DFSM minimization algorithm

- Input a DFSM $M = \langle \Sigma, Q, \delta, q_0, F \rangle$
- Iteratively refine a partition of the set of states, where each set in the partition consists of states so far undistinguishable.
- Start with the partition $\Pi = \{F, Q - F\}$
- Refine the current Π by splitting sets $S \in \Pi$ into sets S_1 and S_2 if there exist $q_1 \in S_1$ and $q_2 \in S_2$ and $a \in \Sigma$ such that
  - $\delta(q_1, a)$ and $\delta(q_2, a)$ are in two different sets of Π.
- Merge sets of undistinguishable states into a single state.

Example: $a(a|0)^*$



A Language for specifying lexical analyzers

$$(0|1|2|3|4|5|6|7|8|9)(0|1|2|3|4|5|6|7|8|9)^*$$
$$(\varepsilon|.(0|1|2|3|4|5|6|7|8|9)(0|1|2|3|4|5|6|7|8|9)^*$$
$$(\varepsilon|E(0|1|2|3|4|5|6|7|8|9)(0|1|2|3|4|5|6|7|8|9)))$$

Descriptional Comfort

Character Classes:

Identical meaning for the DFSM (exceptions!), e.g.,
le = a - z A - Z
di = 0 - 9
Efficient implementation: Addressing the transitions indirectly through an array indexed by the character codes.

Symbol Classes:

Identical meaning for the parser, e.g.,
Identifiers
Comparison operators
Strings

Descriptional Comfort cont'd

Sequences of regular definitions:

$$
\begin{array}{rcl}
A_1 & = & R_1 \\
A_2 & = & R_2 \\
 & \cdots & \\
A_n & = & R_n
\end{array}
$$

Sequences of Regular Definitions

Goal: Separate final states for each definition

1. Substitute right sides for left sides
2. Create an NFSM for every regular expression separately;
3. Merge all the NFSMs using ε transitions from the start state;
4. Construct a DFSM;
5. Minimize starting with partition

$$\{F_1, F_2, \ldots, F_n, Q - \bigcup_{i=1}^{n} F_i\}$$

Flex Specification

Definitions
%%
Rules
%%
C-Routines

Flex Example

```
%{
extern int line_number;
extern float atof(char *);
%}
DIG      [0-9]
LET     [a-zA-Z]
%%
[=#<>+-*]          { return(*yytext); }
({DIG}+)  { yylval.intc  = atoi(yytext); return(301); }
({DIG}*\.{DIG}+(E(\+|\-)?{DIG}+)?)
           {yylval.realc = atof(yytext); return(302); }
\"(\\.|[^\"\\])*\" { strcpy(yylval.strc, yytext);
                     return(303); }
"<="               { return(304); }
:=                 { return(305); }
\.\.               { return(306); }
```

Flex Example cont'd

```
ARRAY                  { return(307); }
BOOLEAN                { return(308); }
DECLARE                { return(309); }
{LET}({LET}|{DIG})* { yylval.symb = look_up(yytext);
                      return(310); }
[ \t]+                { /* White space */ }
\n                    { line_number++; }
.                     { fprintf(stderr,
   "WARNING: Symbol '%c' is illegal, ignored!\n", *yytext);}
%%
```