Exercise Sheet 6

Compiler Design Lab
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Exercise 6.1 Dominance and Data Flow

Dominance is defined for directed graphs as follows:

Let (N, E, s) be a directed graph with nodes N, edges $E \subseteq N \times N$ and a unique starting node $s \in N$ with no incoming edges. A node $n_1 \in N$ *dominates* a node $n_2 \in N$ in (N, E, s), if for every path π from s to n_2, n_1 occurs on π.

Develop a data flow analysis to compute the set of dominators of each block in a control flow graph. The analysis is performed on a control flow graph (CFG), which only consists of basic blocks and control flow edges between them. (The actual instructions do not matter for dominance.) Further, a CFG has a unique start node. A basic block is a maximal sequence of instructions which starts with a label and ends in a conditional or unconditional branch and does not contain any other label or branch. Consider the following aspects:

- What is the domain, which is a complete lattice, of the analysis? In particular, describe $\bot$ and $\top$.
- What is the join operator $\sqcup$?
- If a block A is dominated by a block B (and $A \neq B$), then all direct predecessors of A are also dominated by B.
- Each block dominates itself.
- What does it mean that information is (un-)safe for this analysis?
- Is the analysis performed forwards (along the control flow) or backwards (against the control flow)?
- What is the initialization at each block?
- What is the set of dominators of an unreachable block?

Draw the CFG for the following program and perform dominance analysis. Label each basic block with an uppercase letter.

```
void f(void) {
  if (...) {
    while (...) {
      if (...)
        break;
    }
  } else {
  }
}
```

Exercise 6.2 Chaotic Iteration

For computing the least fixed-point of the abstract semantics of a program, we have shown the correctness of the Kleene iteration scheme. Following this scheme, if we have an analysis that computes information for each program point, we have to apply the transformers for all program locations in parallel in each step (starting from $\bot^n$).

In the lecture, we claimed that we can instead apply transformers for individual program locations independently in an arbitrary order and still obtain the same least fixed-point as Kleene iteration. This exercise proves the validity of this claim.

First some definitions and notations:
Let $(L, \sqsubseteq)$ be a complete lattice with no infinite ascending chains. We extend the domain of $\sqsubseteq$ to n-tuples of elements of L in a straight-forward way: $\underline{a} \sqsubseteq \underline{b} \equiv \forall i \in \{1, \ldots, n\}.\ a_i \sqsubseteq b_i$.

Let $F : L^n \to L^n$ with $F = (f_1, \ldots, f_n)$. Let further all $f_i : L^n \to L$ be monotone. (From this also follows that F is monotone.) For convenience, we lift the component functions f_i to return n-tuples:

$$\widetilde{f_i} : L^n \to L^n,\ \widetilde{f_i}(\underline{x}) = (x_1, \ldots, x_{i-1}, f_i(\underline{x}), x_{i+1}, \ldots, x_n)$$

Note that the monotonicity of the $\widetilde{f_i}$ follows from the monotonicity of the f_i.

We add a shorthand notation for compositions of these functions: Let $\sigma \in \{1, \ldots, n\}^*$ be a sequence of indices. We define:

$$\widetilde{f_\sigma} = \begin{cases} \lambda x.\ x & \text{if } \sigma = \varepsilon \\ \widetilde{f_k} \circ \widetilde{f_{\sigma'}} & \text{if } \sigma = k\sigma' \end{cases}$$

Proceed as follows to prove the correctness of the chaotic iteration scheme:

1. Prove that for any sequence ρ of indices, the intermediate results of $\widetilde{f_\rho}$ form an ascending chain:

   $$\forall \sigma \in \{1, \ldots, n\}^*.\ \forall k \in \{1, \ldots, n\}.\ \widetilde{f_\sigma}(\bot^n) \sqsubseteq \widetilde{f_{k\sigma}}(\bot^n)$$

   Hint: Use an induction over the length of σ.

2. When performing our chaotic iteration, we apply component transformers $\widetilde{f_i}$ to $\bot^n$ until no application of any $\widetilde{f_i}$ can change the the result anymore. Argue that chaotic iteration terminates and computes a fixed-point of F.

3. Prove that the fixed-point x of F that we compute via chaotic iteration is the least fixed-point of F.
   Hint: Compare the intermediate values of applying $\widetilde{f_\sigma}(\bot^n)$ to those of applying $F^{|\sigma|}(\bot^n)$.

Project task E Semantic Analysis

Implement semantic analysis.

- You can perform this either during parsing and AST construction or as a separate phase.
- Semantic analysis augments `--parse` and `--print-ast`.
- Major parts are name and type analysis. Name analysis associates identifiers with declarations. Type analysis associates expressions with types. It encompasses the Constraints and Semantics clauses.
- If you delayed certain syntactic checks (e.g. rejecting `a || b = c`), perform them now.
- The only *null pointer constant*, which you need to support, is literal `0`.
- The previous restriction and the restricted language subset ensure that it is not necessary to evaluate the value of any expressions during semantic analysis.
- It is not necessary to accept programs which contain functions that return a struct or have one as parameter. Similarily, it is not necessary to accept programs which contain assignments of struct type.
- It is not necessary to accept programs which contain anonymous structs.
- You do not need to handle `__func__`.
- Use `int` for the types `ptrdiff_t` and `size_t`.
- Due to the restricted language subset, type compatibility degenerates to equality.
- For the error location use the location of the (first) terminal of the syntactic construct where the error was detected. E.g. for adding two pointers, show the location of the `+`. For an `if` whose condition is not scalar, show the location of the keyword `if`.
- If you are uncertain about some aspect, ask!
- The soft deadline for this milestone is 2017-12-22.
- Keep it simple!