Exercise Sheet 3

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Polyhedral Analysis

Exercise 3.1: 6 points

1. Give the inequalities for the following frame (2P):

$$S = \{(1,1),(1,2),(2,3)\}, R = \{(1,1),(1,0)\}, D = \emptyset$$

2. Consider the example polyhedron at the end of Section 3.2 of the paper by Halbwachs & Cousot[1]. Give the resulting polyhedron (frame and constraints) after each of the following assignments.
   (a) $x_1 := x_2$ (1P)
   (b) $x_1 := 1$ (1P)
   (c) $x_1 := x_1 + 1$ (2P)

Exercise 3.2: 6 points

1. Consider a polyhedron P given by its inequalities $Ax \leq b$. Derive the inequalities of the convex hull $P' = A'x \leq b'$ of P and a vertex s. Note that a point x' is in P' if and only if it is a convex combination of the vertices of a point in P and s (3P).
2. Compute the convex hull of the polyhedron from the end of Section 3.2 and the vertex (4, 6) and eliminate λ from the system of inequalities (3P).

Exercise 3.3: (Bonus Question, 2 points)

Let P by the set of all polyhedra over $\mathbb{R}^n$. Can the concretization function

$$\begin{aligned} \gamma : P &\rightarrow \mathcal{P}(V \rightarrow \mathbb{R}) \\ (A, b) &\mapsto \{v_1 \mapsto x_1, \ldots, v_n \mapsto x_n \mid A[x_1 \cdots x_n]^T \leq b\} \end{aligned}$$

be completed (by a function α) to a Galois connection? Explain.

[1] Halbwachs & Cousot: Automatic Discovery of Linear Restraints Among Variables of a Program, `http://www.di.ens.fr/~cousot/COUSOTpapers/publications.www/CousotHalbwachs-POPL-78-ACM-p84--97-1978.pdf`